ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Homework Assignment #1 – due via Moodle at 11:59 pm on Thursday, Jan. 25, 2024
[Prob. 5 revised 1/23/24]

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That's okay. If you justify any approximations you make, you will be given full credit for such answers.

The constitutive parameters (ε, μ, and σ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8th ed.).

Note that the first set of problems will be graded and the second set will not be graded. Only the graded problems must be submitted by the deadline above; do not submit the ungraded problems.

***Graded Problems*:**

1. Find approximate values for the capacitance and inductance per unit length (C' and L' in F/m and H/m, respectively) of RG-11A coaxial cable operated at a frequency of 50 MHz using the characteristic impedance and dielectric type information available at the "RF Cafe's Coaxial Cable Specifications Chart" link at the course web site. Verify that your value for C' is close to the one given in the chart. *Hint*: Make an educated guess regarding whether the line can be assumed to have low loss.

2. A low-loss ($R' << \omega L'$ and $G' << \omega C'$) transmission line with polyethylene insulation is tested at a frequency of 100 MHz. The characteristic impedance is found to have a value of $50.5 - j0.0822\ \Omega$, the attenuation constant is found to be 0.00523 Np/m, and the wavelength along the line is 2.00 m. Using this information alone, estimate the line parameters R', L', G', and C'. ($G' \neq 0$ in this case.) *Hints*: Recall that $\beta = 2\pi/\lambda$ and that $\gamma = \alpha + j\beta$. Also, consider how you might use the product or ratio (or both) of the exact expressions for Z_0 and γ.

3. The following equation expresses the forward voltage wave along a section of slightly lossy two-wire line in phasor form. The line has a matched load (i.e., $Z_L = Z_0$) and the characteristic impedance $Z_0 = 300\ \Omega$:

$$\tilde{V}(z) = 4.3e^{-j0.33\pi} e^{-0.000168z} e^{-j0.0455z}\ \ \mu\text{V},$$

where $z = 0$ is the location of the load. Assuming that the loss in the dielectric (insulation) is negligible (i.e., $G' = 0$), find the values of the resistance per unit length R' of the wire and the relative permittivity (also called the "dielectric constant") ε_r of the dielectric. The frequency of operation is 1.5 MHz.

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4. Convert the phasor expression in the previous problem to time-domain form at the frequency 1.5 MHz.

5. **[revisions in boldface]** For low-loss transmission lines operated within their intended range of frequencies, the line parameters R' and G' are small enough so that $R' << \omega L'$ and $G' << \omega C'$. Thus,

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{R'G' + j\omega R'C' + j\omega G'L' - \omega^2 L'C'}$$
$$\approx \sqrt{j\omega R'C' + j\omega G'L' - \omega^2 L'C'}$$

The last approximation can be made because the relationships $R' << \omega L'$ and $G' << \omega C'$ together imply that both $R'G' << \omega G'L'$ and $R'G' << \omega R'C'$. From this point, use additional mathematical manipulations and the approximation $(1 \pm x)^{1/2} \approx 1 \pm x/2$, which is valid for $|x| << 1$ whether x is real, imaginary, or complex, to show that the attenuation and phase constants can be respectively approximated as

$$\alpha \approx \frac{R'}{2Z_o} + \frac{G'Z_o}{2} \quad \text{and} \quad \beta \approx \omega\sqrt{L'C'}\,.$$

Substitute your estimates of the line parameters R', L', G', and C' found in **Graded Prob. 2** and the approximate purely real value of Z_0 from that problem into these formulas, and verify that they give numerical values for α and β that are very close to those given or derived in **Graded Prob. 2**.

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. As indicated in Table 2-2 of the textbook (Ulaby and Ravaioli, 8th ed.), the characteristic impedance of a coaxial line depends on the ratio of the outer conductor radius to the inner conductor radius (b/a). For a 50 Ω line, compare the loss per unit length in dB/m at 200 MHz for a 50 Ω line with an outer conductor radius of 2.5 mm to one with an outer radius of 5.1 mm. The outer to inner radius ratio (b/a) must be the same for both to maintain the 50 Ω characteristic impedance. The conductors are copper, and the insulation is polyethylene. Assume that the operating frequency is low enough that the conductance per unit length (G') can be ignored.

2. An 8.2 m long coaxial transmission line with a labeled characteristic impedance of 50 Ω but unknown insulation is to be used in an application in which it will carry signals with frequencies of 30 MHz or less. Measurements made at 10 MHz reveal that the end-to-end (i.e., over the full cable length) signal loss is 0.21 dB and the end-to-end phase shift with a matched load ($Z_L = Z_0$) is 123°. At the test frequency, the loss due to current leakage though the dielectric can be considered negligible (i.e., $G' = 0$). Estimate the values of R', L', and C' for the line. *Hint*: The loss is dB/m is equal to 8.68 times the loss in Np/m (See Sec. 7-6.3 in the textbook).

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3. The components and copper traces on a printed circuit board must be laid out carefully if the circuit is to be operated at high frequencies. Long circuit traces might need to be treated as transmission lines. For the radio services listed below, determine whether or not the electrical size (i.e., its dimensions in wavelengths) of a circuit board that could fit inside a wristwatch of roughly 1.5 cm diameter is large enough to require transmission line analysis for traces that run the full diameter of the board. Assume that the equivalent permeability and permittivity of the circuit board substrate material are $\varepsilon = 4\varepsilon_o$ and $\mu = \mu_o$, respectively, where ε_o and μ_o are the free-space values. Show your work, and for each case justify your conclusion. Base your calculations on the center frequencies allocated to each service by the United States Federal Communications Commission. Cite the source of your information for those cases in which the operating frequency is not explicitly given below.

WKOK-AM radio, Sunbury, PA (transmission facility on County Line Rd. off Rt. 15)
Lowest proposed 5G wireless telecommunications band (roughly 28 GHz)
Bluetooth wireless link standard (estimate freq. if necessary)
Amateur radio "6-meter" band

4. Find the phasors that correspond to the following functions of time and space. Also, find the frequency (linear, not radian; i.e., *f*, not ω) at which each phasor is valid. Watch the units!

a. $i(y,t) = 7.3e^{-0.05y}\cos\left(8\times10^{6}t + 12y\right)$ mA
b. $v(x,t) = 15\cos\left(87{,}500\pi t - 1.2\pi x\right)$ mV
c. $i(z,t) = 45e^{-0.11z}\cos\left(8\times10^{9}t\right)$ mA

5. A 10 m long section of "zip cord" (the two-conductor type of power cord used to connect electrical equipment, such as desk lamps, to AC power sockets) consists of two #18 AWG wires embedded in PVC insulation. If the wires are 2.5 mm apart, and if PVC has a permittivity of $3.4\varepsilon_o$ at HF (3–30 MHz) and below, estimate the characteristic impedance of the zip cord in that frequency range. Assume that the effective permittivity of the insulation for the purpose of calculating phase velocity is approximately equal to the average value of PVC and the surrounding air. Also assume that the line is lossless. (The latter is a BIG assumption!) PVC is a non-magnetic material, which means that $\mu = \mu_o$.

6. The lumped-element model given in the textbook (Ulaby and Ravaioli, 7th ed.) is not the only one that can be used to determine the voltages and currents along transmission lines. A valid alternative model is shown in the diagram below. In this case, the series resistance and inductance are split between the two conductors. Although this is a more realistic model than the one described in the textbook, the extra complexity is not really necessary. Show that the telegrapher's equations in phasor form given in Equation (2.18) of the textbook are obtained using the modified lumped-element model depicted below.