ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Homework Assignment #2 – due via Moodle at 11:59 pm on Thursday, Feb. 1, 2024
[deadline extended]

***Instructions, notes, and hints*:**

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. If you justify any approximations that you make, you will be given full credit for such answers.

The constitutive parameters (ε, μ, and σ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8th ed.).

Note that the first six problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Consider the following expression for the total phasor voltage along a lossless 50 Ω transmission line that has polyethylene insulation (ε_r = 2.25). Forward and reflected waves are both present.

$\tilde{V}(z) = 0.80e^{-j0.48\pi}e^{-j0.22z} - 0.13e^{j0.75\pi}e^{j0.22z}$ mV, where $z = 0$ is the location of the load.

Find numerical values for the following quantities:

a. the frequency of operation in MHz.
b. the wavelength in the dielectric.
c. the complex coefficient V_0^+ of the forward (+z propagating) voltage wave.
d. the complex coefficient I_0^- of the reflected (−z propagating) current wave.
e. the phasor representation (in polar form) of the total voltage at the load's location.
f. the phasor representation (in polar form) of the total voltage at $z = -20$ m.
g. the phase shift between the total voltages calculated in parts *e* and *f*.

2. A CATV (cable television) system "headend" amplifier is connected to an HDTV via a 10 m long lossless 75 Ω coaxial transmission line that has polyethylene insulation. The equivalent input impedance of the HDTV (which acts as the load on the coax) is 100 – j30 Ω at the operating frequency of 330 MHz. Assuming that the HDTV is located at z = 0, the forward voltage wave coefficient is $V_0^+ = 0.10\ \angle\ 54°$ V (i.e., the phase is 54°). Find the numerical value of the *total* voltage at the input end of the line, at the load, and 2.0 m from the load. Express each voltage as a phasor in polar form and as a time-domain function.

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3. Show that the magnitude of the total voltage along a lossy transmission line (one for which $\alpha \neq 0$) can be expressed as

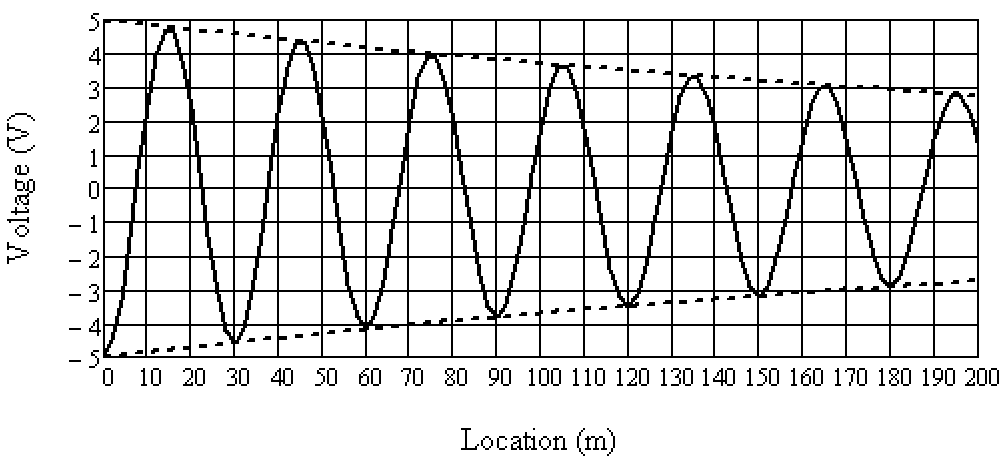
$$\left|\tilde{V}(z)\right| = \left|V_0^+\right| \left\{ e^{-2\alpha z} + |\Gamma|^2 e^{2\alpha z} + 2|\Gamma|\cos\left(\theta_r + 2\beta z\right) \right\}^{1/2}.$$

Hint: Begin with the general formula for the voltage along a lossy line,

$$\tilde{V}(z) = V_0^+ \left(e^{-\gamma z} + \Gamma e^{\gamma z}\right) = V_0^+ \left(e^{-(\alpha + j\beta)z} + \Gamma e^{(\alpha + j\beta)z}\right) = V_0^+ \left(e^{-\alpha z} e^{-j\beta z} + \Gamma e^{\alpha z} e^{j\beta z}\right),$$

and apply the polar form of the expression for the reflection coefficient given by $\Gamma = |\Gamma| e^{j\theta_r}$.

4. The waveform shown below depicts a traveling wave propagating along a low-loss (but not lossless) transmission line toward the right. There is no reflected wave. Although it is not shown in the figure, the spatial variable is *x*, with *x* increasing to the right. The line uses polystyrene (not polyethylene) insulation. The dashed line is not part of the waveform; it merely indicates the envelope of the wave. The plot is of the total real voltage along the line at the instant in time t = 10 ns. Find a phasor expression that fully describes the wave, and find the frequency (in MHz) at which the phasor is applicable.



5. Suppose that an unknown but purely real load impedance is connected to one end of a lossless transmission line with a characteristic impedance of 300 Ω. The other end is driven by a signal generator with an output impedance of 300 Ω that is operating at 150 MHz and that has a Thévenin equivalent voltage of 50 mVpk. Measurements reveal that the VSWR along the line is 2.5. Find all of the possible values that the purely real load impedance can have.

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***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Starting with the expression given in Graded Prob. 3 for the voltage magnitude along a lossy line, find an expression for the VSWR along a lossy line as a function of the location z. Make sure that the result is in a physically meaningful form, that is, that it gives a positive value for the VSWR. It should reduce to the one applicable in the lossless case when $\alpha = 0$. Use the VSWR expression to determine the value that the VSWR approaches as the distance from the load becomes very large (i.e., for very large negative values of z). Explain qualitatively what this result implies. That is, explain what is physically happening to the forward and reflected voltage waves on a long line and why that causes the VSWR to approach the asymptotic value at the input end (the end opposite the load).

2. As shown below, a load consisting of a known resistance R_L and an unknown capacitance C_L is connected to the end of a lossless coaxial transmission line with $Z_0 = 50\ \Omega$ and polyethylene insulation. At an operating frequency of 20 MHz, the VSWR along the line is found to be 1.8. Find the value of C_L.