

**Homework Assignment #2 – due via Moodle at 11:59 pm on Monday, Feb. 10, 2025*****Instructions, notes, and hints:***

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That's okay. If you justify any approximations that you make, you will be given full credit for such answers.

The constitutive parameters ( $\epsilon$ ,  $\mu$ , and  $\sigma$ ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8<sup>th</sup> ed.).

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems:***

1. Consider the following expression for the total phasor voltage along a lossless  $50\ \Omega$  transmission line that has polyethylene insulation ( $\epsilon_r = 2.25$ ). Forward and reflected waves are both present.

$$\tilde{V}(z) = 0.80e^{-j0.48\pi}e^{-j0.22z} - 0.13e^{j0.75\pi}e^{j0.22z} \text{ mV, where } z = 0 \text{ is the location of the load.}$$

Find numerical values for the following quantities:

- a. the frequency of operation in MHz.
  - b. the wavelength in the dielectric.
  - c. the complex coefficient  $V_0^+$  of the forward (+ $z$  propagating) voltage wave.
  - d. the complex coefficient  $I_0^-$  of the reflected ( $-z$  propagating) current wave.
  - e. the phasor representation (in polar form) of the total voltage at the load's location.
  - f. the phasor representation (in polar form) of the total voltage at  $z = -20$  m.
  - g. the phase shift between the total voltages calculated in parts e and f.
2. A CATV (cable television) system “headend” amplifier is connected to an HDTV via a 10 m long lossless  $75\ \Omega$  coaxial transmission line that has polyethylene insulation. The equivalent input impedance of the HDTV (which acts as the load on the coax) is  $100 - j30\ \Omega$  at the operating frequency of 330 MHz. Assuming that the HDTV is located at  $z = 0$ , the forward voltage wave coefficient is  $V_0^+ = 0.10 \angle 54^\circ$  V (i.e., the phase is  $54^\circ$ ). Find the numerical value of the *total* voltage at the input end of the line, at the load, and 2.0 m from the load. Express each voltage as a phasor in polar form and as a time-domain function.

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3. Show that the magnitude of the total voltage along a lossy transmission line (one for which  $\alpha \neq 0$ ) can be expressed as

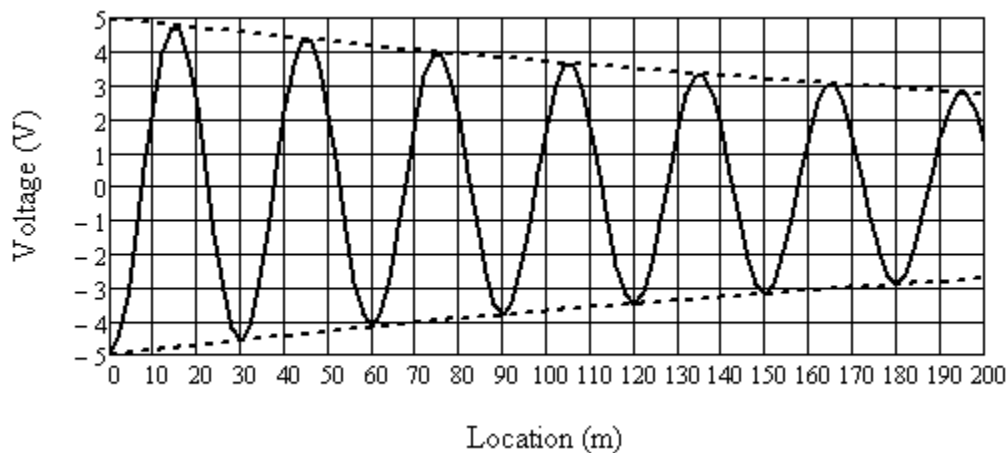
$$|\tilde{V}(z)| = |V_0^+| \left\{ e^{-2\alpha z} + |\Gamma|^2 e^{2\alpha z} + 2|\Gamma| \cos(\theta_r + 2\beta z) \right\}^{1/2}.$$

*Hint:* Begin with the general formula for the voltage along a lossy line,

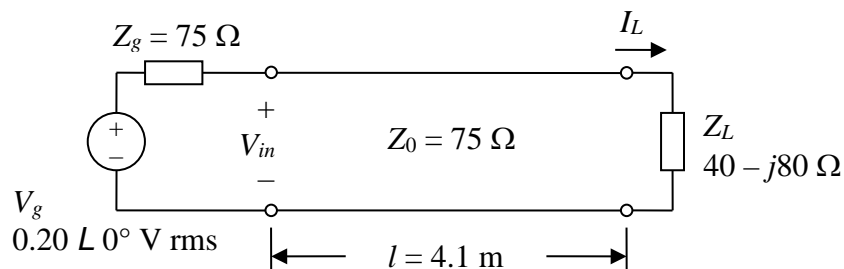
$$\tilde{V}(z) = V_0^+ (e^{-\gamma z} + \Gamma e^{\gamma z}) = V_0^+ (e^{-(\alpha + j\beta)z} + \Gamma e^{(\alpha + j\beta)z}) = V_0^+ (e^{-\alpha z} e^{-j\beta z} + \Gamma e^{\alpha z} e^{j\beta z}),$$

and apply the polar form of the expression for the reflection coefficient given by  $\Gamma = |\Gamma| e^{j\theta_r}$ .

4. The waveform shown below depicts a traveling wave propagating along a low-loss (but not lossless) transmission line toward the right. There is no reflected wave. Although it is not shown in the figure, the spatial variable is  $x$ , with  $x$  increasing to the right. The line uses polystyrene (not polyethylene) insulation. The dashed line is not part of the waveform; it merely indicates the envelope of the wave. The plot is of the total real voltage along the line at the instant in time  $t = 10$  ns. Find a phasor expression that fully describes the wave, and find the frequency (in MHz) at which the phasor is applicable.



5. A 4.1-m section of  $75 \Omega$  coaxial line with polyethylene insulation is driven by a signal source as shown in the figure at the top of the next page. The generator (signal source) voltage expressed as a phasor is  $V_g = 0.20 \angle 0^\circ$  V rms, and the load impedance is  $Z_L = 40 - j80 \Omega$ . Find the phasor input voltage  $V_{in}$  and the phasor load current  $I_L$ . Express both phasors in polar form. The operating frequency is 300 MHz.



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### Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Suppose that an unknown but purely real load impedance is connected to one end of a lossless transmission line with a characteristic impedance of  $300\ \Omega$ . The other end is driven by a signal generator with an output impedance of  $300\ \Omega$  that is operating at  $150\ \text{MHz}$  and that has a Thévenin equivalent voltage of  $50\ \text{mVpk}$ . Measurements reveal that the VSWR along the line is  $2.5$ . Find all of the possible values that the purely real load impedance can have.
2. Starting with the expression given in Graded Prob. 3 for the voltage magnitude along a lossy line, find an expression for the VSWR along a lossy line as a function of the location  $z$ . Make sure that the result is in a physically meaningful form, that is, that it gives a positive value for the VSWR. It should reduce to the one applicable in the lossless case when  $\alpha = 0$ . Use the VSWR expression to determine the value that the VSWR approaches as the distance from the load becomes very large (i.e., for very large negative values of  $z$ ). Explain qualitatively what this result implies. That is, explain what is physically happening to the forward and reflected voltage waves on a long line and why that causes the VSWR to approach the asymptotic value at the input end (the end opposite the load).
3. As shown below, a load consisting of a known resistance  $R_L$  and an unknown capacitance  $C_L$  is connected to the end of a lossless coaxial transmission line with  $Z_0 = 50\ \Omega$  and polyethylene insulation. At an operating frequency of  $20\ \text{MHz}$ , the VSWR along the line is found to be  $1.8$ . Find the value of  $C_L$ .

