Homework Assignment #4 – due via Moodle at 11:59 pm on Thursday, Feb. 22, 2024
[Graded Probs. 5 and 6 deferred to HW #5]

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

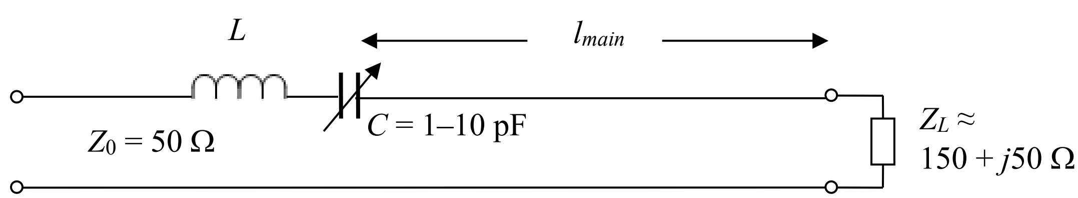
It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first six problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. A 50-Ω microstrip line will be used to supply a wireless broadband signal to an amplifier with an input impedance of $30 + j45\ \Omega$ and that operates at a frequency of 2.45 GHz. The effective relative permittivity of the microstrip line is 4.2. Find the closest location to the load (i.e., the point nearest the amplifier's input terminals) at which an SMT (surface mount technology) capacitor can be inserted in series with the line to achieve an impedance match. Also find the required capacitance value in picofarads.

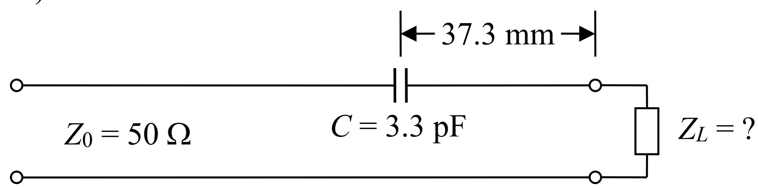
2. As shown below, a load of roughly $150 + j50\ \Omega$ at an operating frequency of 1.6 GHz is connected to a 50 Ω microstrip line with $\varepsilon_r = 4.0$. Because of manufacturing tolerances, the load impedance is likely to vary considerably from the indicated value. An important design goal is to make all parts of the system as small as possible. The experienced RF designers know that the closest point to an inductive load where a series matching element can be placed is likely to have a negative input reactance (X_{in}) and therefore would require an inductor to achieve a match. They would like to use a variable inductor to accommodate the variable load impedance, but they are bulky and expensive and often lossy in the RF range. The design team realizes that they can implement a variable inductance using a fixed inductor in series with a variable capacitor as shown below. (The drawing is not to scale. The dimensions of the LC combination are much less than l_{main}.) Assuming that only variable capacitors with a range of 1–10 pF are available, find the required fixed inductance value L so that the range of adjustability of the LC combination is centered on the required inductive reactance value. Also find the required distance l_{main} from the load in millimeters.



(*continued on next page*)

3. Attempt to design a series-element matching system to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The dimension of the line section may be in wavelengths. Explain conceptually why this task is impossible.

4. Suppose that you are working for a company that has found an old piece of equipment with the microstrip matching system depicted schematically below. The load is an antenna, but there is no documentation that records its input impedance. However, you can measure the dimensions of the microstrip line, and you can read the value of the surface mount capacitor C. You also know that the system operated at 1.6 GHz and that the microstrip line has an effective relative permittivity of $\varepsilon_r = 4.0$. Use the given information to estimate the input impedance of the antenna. (The antenna's input impedance is the load impedance of the transmission line.)



5. **[deferred to HW #5]** Suppose that an IEEE 802.11a WLAN signal source with an output impedance Z_g of 50 Ω (purely real) can supply 2.0 mW of power to a load under matched conditions (i.e., when a 50 Ω load is connected to the source). An amplifier with an input impedance of $42 - j32\ \Omega$ is connected to the signal source through a 50 Ω microstrip transmission line with a length of 1.2λ. The line has an effective relative permittivity of 4.2, and the operating frequency range has a narrow bandwidth centered at 5.65 GHz. You may assume that the line is lossless. Find:

   a. the time-average incident power that flows along the microstrip line.
   b. the time-average reflected power that flows along the microstrip line.
   c. the time-average power delivered by the signal source (represented by the Thévenin equivalent circuit consisting of V_g and Z_g) to the input end of the microstrip line.
   d. the time-average power actually delivered to the mismatched load.

6. **[deferred to HW #5]** Suppose that in the previous problem a series capacitor is inserted into the microstrip line to match the amplifier's input impedance to the 50-Ω line. The capacitor has a value of 0.78 pF and is located 7.4 mm (0.28λ) from the load. How much power is delivered to the load in this case? You might be able to solve this problem with minimal effort, but you must provide some kind of justification for your answer.

(*continued on next page*)

***Ungraded Problems*:**

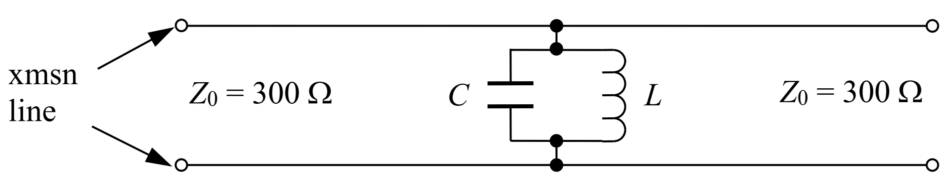
The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Imagine a capacitor and an inductor connected across a transmission line at the same point, as shown below. The components have reactances of equal magnitude but opposite algebraic sign at the operating frequency, so they form a parallel resonant circuit that has an equivalent combined impedance of infinity. That means that the presence of the circuit is not "felt" by waves propagating along the line at the resonant frequency. However, waves at other frequencies will experience partial reflection at the location of the LC circuit, with greater reflection at frequencies farther from resonance. The line and LC circuit therefore form a type of band-pass filter. Although lumped capacitors and inductors can be used, more typically transmission line stubs are employed. Find the electrical lengths (i.e., in wavelengths) of a pair of shunt (parallel) stubs necessary to create reactances of

a. $X_L = |X_C| = 30\ \Omega$
b. $X_L = |X_C| = 3000\ \Omega$,

where X_L is the equivalent inductive input reactance of one stub, and X_C is the equivalent capacitive input reactance of the other stub. Use open-circuited stubs. Both stubs are connected in parallel with the line at the same point (where the inductor and capacitor are connected in the diagram below).

For each case, add the lengths of the two stubs together. Do you notice something interesting? What would the stub structure look like if $X_L = |X_C| \to \infty$?



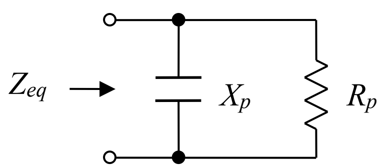
Note: The following problems focus on shunt-element (parallel-element) matching systems. Although this topic has been deemphasized this year, the problems below have been included so that you can see some of the applications of shunt-element matching and the differences in design methodology compared to series-element systems. This topic might be brought back in future offerings of ECEG 390.

2. An antenna with an equivalent input impedance of $250 + j150\ \Omega$ is to be connected to a parallel-wire air-insulated transmission line with $Z_0 = 450\ \Omega$. The operating frequency is 20 MHz. The designers would like to achieve an impedance match by inserting an inductor or capacitor in parallel with the line at an appropriate distance from the load. Determine the location nearest to the load at which each type of component should be inserted. Also specify the required inductance and capacitance values.

(*continued on next page*)

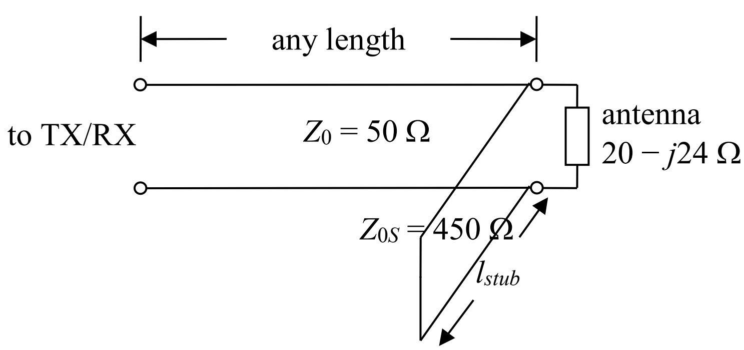
3. Suppose in the previous problem that a shorted stub will be used at each location instead of a lumped reactance. Find the required physical length of each stub. The stubs are to be made from sections of 450 Ω parallel-wire transmission line with air insulation.

4. The formula for l_{main} shown below is used in the design of shunt element-based matching networks. Suppose that a network is to be designed to match a load impedance of $19.5 - j24.4\ \Omega$ to a 75-Ω transmission line. At the distance l_{main} from the load, the equivalent input impedance can be modeled by the parallel equivalent circuit shown below (represented by R_p and X_p; the *p* subscripts are for "parallel"). Find the values of R_p and X_p that constitute an appropriate model for this case. Note that Z_{eq} is the impedance seen looking into the line toward the load at the distance l_{main} from the load. The dimensions of the equivalent circuit below are tiny compared to a wavelength.



$$l_{main} = \frac{\lambda}{4\pi}\left[\theta_r \pm \cos^{-1}\left(-|\Gamma|\right)\right]$$

5. A class of antennas known as Yagi-Uda arrays (after the Japanese engineers who invented the concept in the 1920s) typically have low input impedances relative to commonly used 50 Ω coaxial cable. One way to achieve an impedance match is to adjust one of the parts of the antenna so that the input impedance is complex with a negative imaginary part (capacitive). Then an inductive stub is connected in parallel with the antenna's terminals, and the result is an excellent match. Depicted below schematically is the feedline and input impedance of a Yagi-Uda array that has already been adjusted to have an appropriate capacitive input impedance. Find the required length of a shorted stub that would result in a good match to 50 Ω. Note that the characteristic impedance Z_{0S} ("*S*" for stub) of the line section that makes up the stub is 450 Ω, not 50 Ω; it is made from a short piece of parallel-wire line rather than coax. The frequency of operation is 144 MHz (the amateur radio 2-meter band). This type of impedance matching system is called a "hairpin" match; many examples can be found online by searching the term "hairpin match yagi."



(*continued on next page*)

6. A day-care center wants to implement a closed-circuit television (CCTV) system to monitor the rooms in the facility using old donated TV sets. All of the TV sets will be tuned to old NTSC channel 3 (center frequency of 69 MHz). Rough measurements reveal that the TV receivers have input impedances close to $290 + j50\ \Omega$, but some differ a little from that value. The CCTV distribution system uses $75\ \Omega$ coaxial cable with polyethylene insulation, so a single shunt-stub matching system made out of the same $75\ \Omega$ cable will be added to each TV set. A "T" connector will be used to connect the stub in parallel with the main transmission line. To deal with the unknown input impedances of the TV sets, the length of the stub will be either $\lambda/4$ or $\lambda/2$, and a variable capacitor will be mounted at the end of the stub to provide adjustability. A sketch of the matching system is shown below.

a. Find the shortest distance d from the load at which the stub should be placed.
b. Given that a variable capacitor (not an inductor) must be placed at the end of the stub, determine whether a $\lambda/4$ or $\lambda/2$ stub should be used. Think about how each stub length would transform the capacitive load impedance to its respective input impedance. Find the physical length of the selected stub in cm.
c. Find the center (average) value of the variable capacitor. Variable capacitors have maximum and minimum values, of course, so an actual system would be designed so that the average expected required capacitance falls in the middle of the range. That should allow enough adjustability to achieve a good – but maybe not perfect – impedance match regardless of the variation in the load impedance.