ECEG 390 **Theory and Applications of Electromagnetics** **Spring 2025**

Homework Assignment #4 – due via Moodle at 11:59 pm on Monday, Mar. 3, 2025

***Instructions, notes, and hints*:**

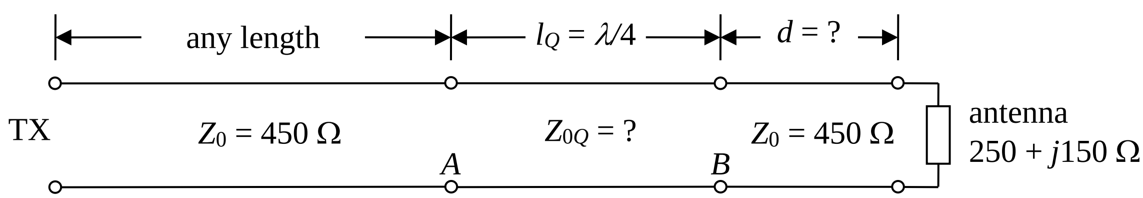
You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use reasonable approximations or assumptions to solve some of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. That could be okay. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Design a quarter-wave section (i.e., find distance the d and the quarter-wave section's characteristic impedance Z_{0Q}) to match an antenna with an impedance of $250 + j150\ \Omega$ at an operating frequency of 20 MHz to a 450 Ω parallel-wire air-insulated transmission line. The matching system is depicted schematically below. The distance d from the antenna to the load side of the quarter-wave section must be as short as possible. Also, find the required physical length l_Q of the quarter-wave section in meters. "TX" is a standard abbreviation for transmitter; it represents the signal source at the input end.



2. For the matching system considered in the previous problem, find:

- **a.** the VSWR along the line between the transmitter and the input of the quarter-wave section (point A).
- **b.** the VSWR along the quarter-wave section.
- **c.** the VSWR between the output of the quarter-wave section (point B) and the load.

3. For the matching system considered in the previous two problems, suppose that a capacitor with a reactance of $-150\ \Omega$ at the operating frequency is inserted in series with the antenna. The quarter-wave matching section is then redesigned. Find the new required distance d, quarter-wave section length l_Q, and characteristic impedance Z_{0Q}.

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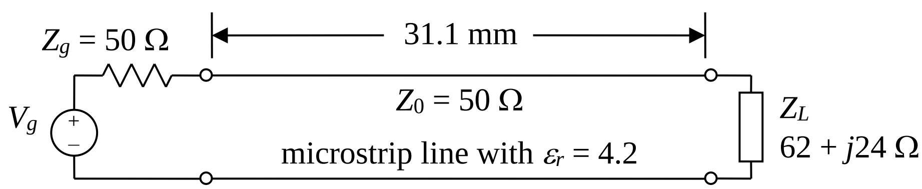
4. Depicted in the diagram below is a WLAN (wireless local-area network) signal source with an output impedance Z_g of 50 Ω (purely real) that can supply 5.0 mW of power to a load under matched conditions (i.e., when a 50 Ω load is connected to the source). An amplifier with an input impedance of 62 + j24 Ω is connected to the signal source through a 50 Ω microstrip transmission line with a length of 1.2λ. The line has an effective relative permittivity of 4.2, and the operating frequency range has a narrow bandwidth centered at 5.65 GHz. Assuming that the line is lossless, find:

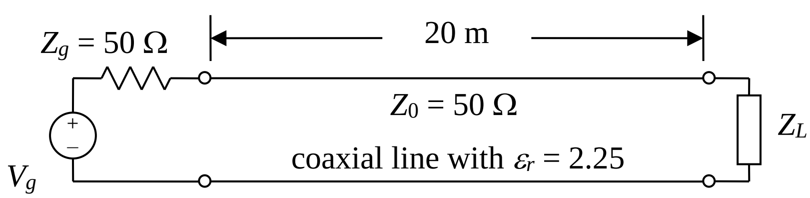
- **a.** the time-average incident power that flows along the microstrip line.
- **b.** the time-average reflected power that flows along the microstrip line.
- **c.** the time-average real power and reactive power delivered by the signal source (represented by V_g and Z_g) to the input end of the microstrip line.
- **d.** the time-average real power and reactive power delivered to the mismatched input port of the amplifier represented by Z_L.
- **e.** the reactive power absorbed or delivered by the microstrip line itself using the expression below**,** where V_0^+ is in peak units. Make sure that the reactive power in the system balances, keeping in mind that there could be some round-off error.

$$Q = \frac{|\Gamma||V_0^+|^2}{Z_0}\left[\sin(\theta_r - 2\beta l) - \sin\theta_r\right]$$



5. Suppose that in the previous problem a series capacitor is inserted into the microstrip line to match the amplifier's input impedance to the 50-Ω line. The capacitor has a value of 1.2 pF and is located 12 mm (0.465λ) from the load. How much power is delivered to the load in this case? You might be able to solve this problem with minimal effort, but you must provide some kind of justification for your answer.

6. In the system shown below with an unknown load impedance Z_L, a power meter inserted in a coaxial transmission line indicates that the incident power flowing toward the load is P^i_{av} = 45 W and the reflected power from the load is P^r_{av} = 5.5 W. The line has polyethylene insulation with ε_r = 2.25. The wavelength within the dielectric is 8.0 m, and the operating frequency is 25 MHz. Find the VSWR along the line.



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***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Attempt to design a quarter-wave line section to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The line and stub dimensions may be in wavelengths. Explain conceptually why this task is impossible.

2. A signal source with an output impedance Z_g of 50 Ω (purely real) and that has an open-circuit output voltage (i.e., Thévenin equivalent voltage) of 2.0 Vpk drives a half-wavelength 50 Ω line. Connected to the 50 Ω line at its load end (i.e., opposite the signal source) is a very long and slightly lossy 100 Ω line. The 100 Ω line is so long that any waves reflected at its far end are so attenuated by the time they return to the junction with the 50 Ω line that they are negligibly weak. See Fig. P2.44 on p. 129 of the textbook (Ulaby and Ravaioli, 7th ed. and 8th ed.) for a visual depiction of the transmission system. Find the time-average incident and reflected power P^i_{av} and P^r_{av} along the 50 Ω line. Also find the power P^t_{av} that is delivered (transmitted, hence the "t" superscript) to the input end of the 100 Ω line. You may assume that the 50 Ω line is lossless.

3. The 1.0 mVpk ideal voltage source and impedance Z_g in the diagram below represent a signal source. Find the maximum available power P_A that the signal source can supply to a load. The three resistors to the right of the source form a 20 dB T-network attenuator. The resistors have tight tolerances (1%), so their values are indicated with three digits of precision. The attenuator is designed to preserve the 50 Ω output impedance of the signal source connected to it. That is, the Thévenin equivalent resistance of the complete circuit to the left of terminals *a*-*b* is 50 Ω. (Verify this for yourself.) Find the available power of the signal source with the attenuator circuit attached, and compare it to the available power without the attenuator. Express the difference between the two power levels in decibels. *Hint*: This is a *20 dB* attenuator.