ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Homework Assignment #5 – due via Moodle at 11:59 pm on Thursday, Mar. 7, 2024
[Prob. 1e revised 3/6/24]

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. If you justify any approximations that you make, you will be given full credit for such answers.

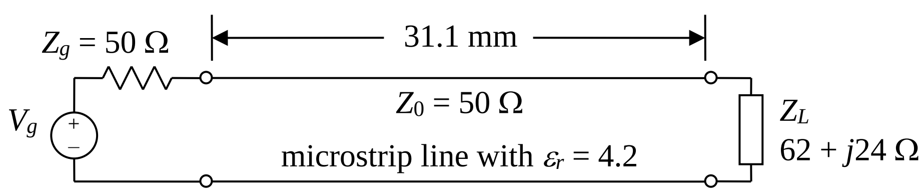
Note that the first six problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Depicted in the diagram below is a WLAN (wireless local-area network) signal source with an output impedance Z_g of 50 Ω (purely real) that can supply 5.0 mW of power to a load under matched conditions (i.e., when a 50 Ω load is connected to the source). An amplifier with an input impedance of $62 + j24$ Ω is connected to the signal source through a 50 Ω microstrip transmission line with a length of 1.2λ. The line has an effective relative permittivity of 4.2, and the operating frequency range has a narrow bandwidth centered at 5.65 GHz. You may assume that the line is lossless. Find:

- **a.** the time-average incident power that flows along the microstrip line.
- **b.** the time-average reflected power that flows along the microstrip line.
- **c.** the time-average real power and reactive power delivered by the signal source (represented by V_g and Z_g) to the input end of the microstrip line.
- **d.** the time-average real power and reactive power delivered to the mismatched input port of the amplifier represented by Z_L.
- **e.** the reactive power absorbed or delivered by the microstrip line itself using the expression below**, where V_0^+ is in peak units**; make sure that the reactive power in the system balances.

$$Q = \frac{|\Gamma|\left|V_0^+\right|^2}{Z_0}\left[\sin\left(\theta_r - 2\beta l\right) - \sin\theta_r\right]$$ **[2 removed from numerator]**



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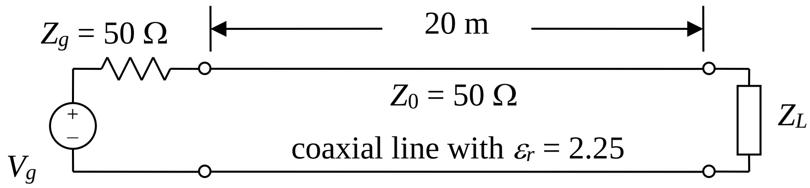
2. Suppose that in the previous problem a series capacitor is inserted into the microstrip line to match the amplifier's input impedance to the 50-Ω line. The capacitor has a value of 1.2 pF and is located 12 mm (0.465λ) from the load. How much power is delivered to the load in this case? You might be able to solve this problem with minimal effort, but you must provide some kind of justification for your answer.

3. In the system shown below, a power meter inserted in a coaxial transmission line indicates that the incident power flowing toward the load is $P^{inc} = 45$ W and the reflected power from the load is $P^{ref} = 5.5$ W. The line has polyethylene insulation with $\varepsilon_r = 2.25$. The wavelength within the dielectric is 8.0 m, and the operating frequency is 25 MHz. Find the VSWR along the line.



4. The differential form of Ampère's law in the time domain is given below. Use dimensional analysis to show that all three terms in the equation have the same units. *Hint*: The vector curl operation is essentially a linear combination of spatial derivatives.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

5. A 0.01λ-long Hertzian dipole is located at the origin of a coordinate system, is oriented along the z-axis, and operates at a frequency of 10 MHz. The input current I_o is $10 \angle 0°$ A. Evaluate the R and θ-components of the phasor electric field (near field) radiated by the antenna in the $\theta = 30°$ direction at the distances a) $R = 0.1\lambda$, b) $R = 1\lambda$, and c) $R = 10\lambda$. You might want to use Matlab, Mathematica, or some other computational software to solve this problem. If you do, remember to include a print-out of your script or program session to submit with your homework. Also consider showing some intermediate calculations in case your answer is incorrect; it might help you receive some partial credit.

6. Evaluate the far electric field approximation for the Hertzian dipole described in the previous problem at the distances a) $R = 0.1\lambda$, b) $R = 1\lambda$, and c) $R = 10\lambda$. Some of these distances are obviously not in the far field, but evaluate them anyway using the far-field expression. Compare your results to parts a, b, and c of the previous problem, and explain why the values you obtained for each part are or are not close to each other. Again, you might want to use software to solve this problem.

(*Ungraded problems begin on the next page.*)

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. A signal source with an output impedance Z_g of 50 Ω (purely real) and that has an open-circuit output voltage (i.e., Thévenin equivalent voltage) of 2.0 Vpk drives a half-wavelength 50 Ω line. Connected to the 50 Ω line at its load end (i.e., opposite the signal source) is a very long and slightly lossy 100 Ω line. The 100 Ω line is so long that any waves reflected at its far end are so attenuated by the time they return to the junction with the 50 Ω line that they are negligibly weak. See Fig. P2.44 on p. 129 of the textbook (Ulaby and Ravaioli, 7th ed. and 8th ed.) for a visual depiction of the transmission system. Find the time-average incident and reflected power P^i_{av} and P^r_{av} along the 50 Ω line. Also find the power P^t_{av} that is delivered (transmitted, hence the "t" superscript) to the input end of the 100 Ω line. You may assume that the 50 Ω line is lossless.

2. For each of the following media, explain whether it should be considered "source-free" or "source filled" in the macroscopic sense. Remember that a source can be either a current (**J**) or charge (ρ_v) distribution. Note that individual static charges in the immediate vicinity of equal and opposite charges are not considered sources. For example, a water molecule has a negative charge center on the oxygen side and two positive charge centers on the hydrogen side; however, the total numbers of electrons and protons in the molecule are equal and the resulting electric field is tiny in extent. Non-ionized water is therefore considered source-free in the macroscopic sense even though individual molecules can be considered sources (static electric dipoles) in the microscopic sense. This problem is intended to be a thought exercise.

- **a.** the sun
- **b.** the dielectric layer of a printed circuit board
- **c.** the copper part of household electrical wiring
- **d.** aluminum lightning rod as a thunderstorm is approaching
- **e.** aluminum lightning rod on a nice day with high humidity
- **f.** cup of distilled water
- **g.** cup of salt water
- **h.** copper traces that constitute a computer's RAM address bus – power on
- **i.** copper traces that constitute a computer's RAM address bus – power off

3. If a Hertzian dipole is aligned along the *z*-axis in the Cartesian coordinate system, then maximum radiation in the far field occurs in all directions in the $\theta = 90°$, $\phi = 0°$ to 360° plane (i.e., the plane perpendicular to the antenna) in the spherical coordinate system. Find the values of θ and ϕ that define the two cones that form the boundary between where the far-field radiated power density is less than and greater than 10% of its maximum value at a given distance R from the antenna.

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4. Equations (9.8b) and (9.8c) in the textbook (Ulaby and Ravaioli, 7^{th} ed. and 8^{th} ed.) give the *R* and *θ*-components, respectively, of the electric field radiated by a Hertzian dipole centered at the origin of the coordinate system and oriented along the *z*-axis. The expressions are shown below. In the far-field approximation, the terms involving $(kR)^2$ and $(kR)^3$ are considered to be insignificant in magnitude and therefore are ignored. Determine the distance *R* in wavelengths that an observer must be from a Hertzian dipole so that the relative magnitudes of the $1/(kR)^2$ and $1/(kR)^3$ terms are 1.0% or less than that of the $1/kR$ term.

$$\tilde{E}_R = \frac{2I_o l k^2}{4\pi}\eta_o e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta$$

$$\tilde{E}_\theta = \frac{I_o l k^2}{4\pi}\eta_o e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta\,.$$

5. Starting with the expressions given in Equations 9.9a and 9.9b in the textbook for the far field of a Hertzian dipole, find the *time-domain* electric and magnetic fields [that is, $\mathbf{E}(R, \theta, \phi, t)$ and $\mathbf{H}(R, \theta, \phi, t)$] at the points listed below. The dipole has the following characteristics: peak input current = 12 A; length = 1.0 m; frequency = 1.0 MHz; surrounded by free space; centered on the coordinate system origin; oriented along the *z*-axis.

a. at $(R, \theta, \phi) = (10{,}000, 30°, 0°)$
b. at $(R, \theta, \phi) = (10{,}075, 30°, 0°)$

Compare the magnitude and phase obtained at each point, and comment on the significance of the comparison.