ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Homework Assignment #6 – due via Moodle at 11:59 pm on Friday, Mar. 22, 2024

***Instructions, notes, and hints*:**

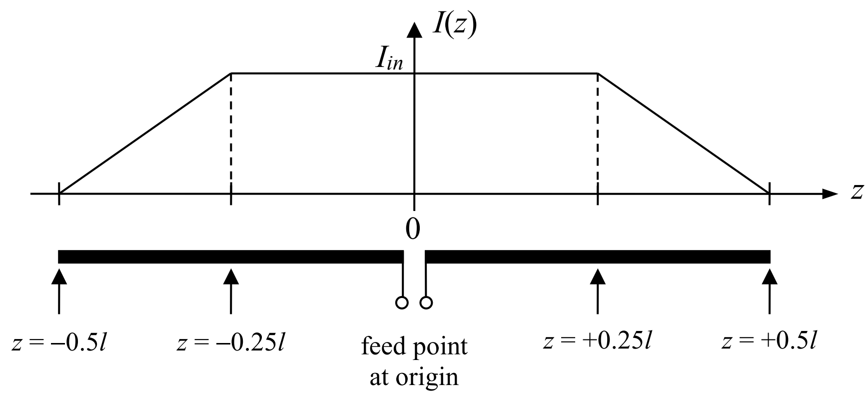
Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. An electrically short ($l << \lambda$) center-fed dipole antenna operating at 10 MHz has an unusual design feature that causes the current distribution along its length to have the functional dependence shown below. The input current at the feed point is I_{in}. The current magnitude stays constant over the middle half of the antenna, and it tapers linearly to zero at the ends over the outer sections. Derive an expression for the far electric field radiated by the antenna, and then determine the directivity of the antenna. The current does not experience a phase shift along the antenna because it is a standing wave; therefore, I_{in} can be assumed to have a real value, and $I(z)$ has a constant phase of 0° along the antenna. (That is, the current is a real-valued function of z.) Assume that the tiny gap at the feed point infinitesimally narrow. The antenna is made from aluminum rod stock with a diameter of 4.8 mm (about 3/16 in).



2. For the antenna in the previous problem, derive an expression for the radiation resistance as a function of the length l. Obtain a numerical value for $l = 0.1\lambda$, and compare it to the radiation resistance of a standard short dipole (with a triangular current distribution) and that of a Hertzian dipole (uniform distribution) of the same length.

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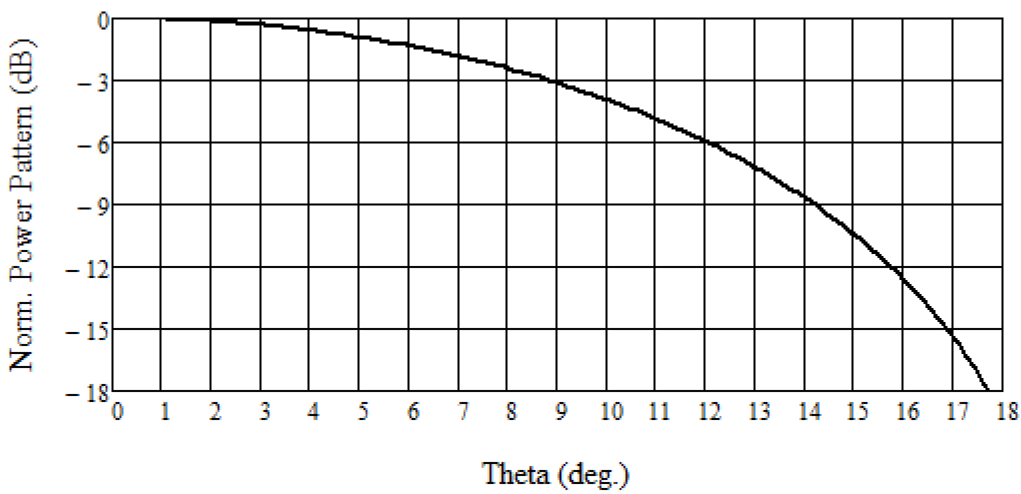
3. A terrestrial (land-based) data link system uses a transmitter connected to a Yagi-Uda antenna that has a gain of 16 dBi. Tests reveal that the received signal's power level is –137 dBm. (The dBm unit represents power referenced to 1 mW; in this case, –137 dBm = 0.0200 fW, or 2.00×10^{-17} W.) However, if the data in the signal is to be detected reliably, the received signal level must be at least –133 dBm (0.0501 fW). The received power level is proportional to the radiated signal's power density in the vicinity of the receiving antenna. A transmitter with a higher output power would be too expensive, so the design team decides to replace the original transmitting antenna with another one with higher gain. Find the minimum required gain (in dBi) of the new antenna.

4. A microwave link antenna has the *normalized* power pattern $F(\theta, \phi)$ shown below. The pattern is rotationally symmetric about the z-axis; that is, there is no ϕ-dependence. Note, however, that the pattern plot is in terms of dB relative to the maximum value of $F(\theta, \phi)$, not as a multiplying factor. Negligible power is radiated for $\theta > 20°$. The maximum directivity of the antenna is 21.3 dBi (in the $\theta = 0$ direction), so the figure below gives the directivity relative to 21.3 dBi. The antenna's efficiency is 80%, the transmitter feeding the antenna has an output power of 20 W, and the transmission line between the transmitter and antenna is essentially lossless. The receiving station is 35 km away, and the system operates at a frequency of 12 GHz. A violent storm forces the antenna out of its proper orientation so that the receiving site is 15° away from the direction of maximum gain. Find the power density (in W/m^2) produced in the vicinity of the receiving station.



5. Suppose that the normalized power pattern of a certain antenna is given by the expression below. Confirm that $F(\theta, \phi)$ has a maximum value of one over all values of θ and ϕ, and then find the directivity of the antenna by analytical (i.e., manual or "on-paper") evaluation. You may check your answer using Matlab, Mathematica, or your calculator to perform the integration numerically, but you must evaluate the integral analytically to receive full credit for this part. You may use an integral table.

$$F(\theta, \phi) = \begin{cases} \left|\sin^2\theta\cos^2\phi\right|, & 0 \le \theta \le 180°, -90° \le \phi \le 90° \\ 0, & 0° \le \theta \le 180°, 90° \le \phi \le 270° \end{cases}$$

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***Ungraded Problem*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Consider the normalized power pattern of the antenna in Graded Problem 5, which is given by

$$F(\theta, \phi) = \begin{cases} \left|\sin^2\theta\cos^2\phi\right|, & 0 \le \theta \le 180°, -90° \le \phi \le 90° \\ 0, & 0° \le \theta \le 180°, 90° \le \phi \le 270° \end{cases}$$

- **a.** Use Matlab to plot the normalized power pattern for $0° \le \theta \le 180°$ in the *xz*-plane (that is, the $\phi = 0°$ half-plane; note that the pattern is zero for $\phi = 180°$). You may use a rectangular plot. Confirm that $F(\theta, \phi)$ has a maximum value of one, and determine the angle of maximum radiation in this plane.
- **b.** Use Matlab to plot the normalized power pattern for $0° \le \phi \le 180°$ in the *xy*-plane (the $\theta = 90°$ plane). You may use a rectangular plot. Confirm that $F(\theta, \phi)$ has a maximum value of one, and determine the angle of maximum radiation in this plane.

2. A large class of reflector antennas have normalized power patterns that can be approximated by the sinc function, where sinc(x) = sin(x)/x. Consider a reflector with a pattern described by the expression given below

$$F(\theta) = \begin{cases} \left|\dfrac{\sin(10\theta)}{10\theta}\right|^2, & \theta \le \dfrac{\pi}{10} \\ 0, & \text{elsewhere} \end{cases}$$

Note that the pattern is rotationally symmetric about the z-axis, so it is not a function of ϕ. Find the direction of maximum radiation, and estimate or calculate the directivity (in dBi) of the antenna using a calculator or software to perform numerical integration. (You may use one method to check the other; this approach is recommended.) Be careful with the special case at $\theta = 0$; consider applying L'Hospital's differentiation rule at that angle.