

Selected Answers to HW #5

Include explanatory text and intermediate calculations in your solutions. You will not receive credit for merely repeating an answer given here without supporting work.

If an answer is not provided below, it is either because the solution is trivial or because disclosure of the answer would reveal too much of the solution.

It is possible that one or more answers are incorrect because of the rush to post them. If you suspect that an answer below is incorrect, please let me know as soon as possible.

$$1. \quad u(x, t) = A_0 e^{-ht} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-\left(\frac{kn^2\pi^2}{L^2} + h\right)t}$$

$$\text{with } A_0 = \frac{1}{L} \int_0^L f(x) dx \quad \text{and} \quad A_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$2. \quad u(x, t) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-kn^2\pi^2 t/L^2} \quad \text{with } A_0 = 100 \text{ and } A_2 = -100$$

$$3. \quad u(x, t) = \sum_{n=1}^{\infty} A_n \left[e^{-\beta t} \cos(q_n t) + \frac{\beta}{q_n} e^{-\beta t} \sin(q_n t) \right] \sin(nx), \quad \text{where } q_n = \sqrt{n^2 - \beta^2},$$

$$\text{with } A_n = 0.2 \int_0^{10} f(x) \sin(nx) dx$$