ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Friday, Aug. 25

1. Basic problems and computations in linear algebra: $A\mathbf{x} = \mathbf{b}$ [review]

   a. A, $\mathbf{x}$ given: geometric transformations (images, outputs from inputs)
   b. A, $\mathbf{b}$ given: system solution (inputs from outputs)
   c. Size & shape matter in defining the solution

2. Solution of $A\mathbf{x} = \mathbf{b}$ using the inverse. For an $N \times N$ (square) system, the following statements are equivalent for the purpose of determining the solvability of the problem.

   a. $A\mathbf{x} = \mathbf{b}$ has a unique solution
   b. A has a unique inverse (A^{-1})
   c. A is non-singular
   d. A has full rank (i.e., rank(A) = N)
   e. det(A) = |A| $\neq 0$

3. Route to finding solutions (implicit inverse computation)

   a. Process: for augmented matrix and reduce (transform) a system to an easier-to-solve form
   b. $A\mathbf{x} = \mathbf{b}$ becomes $U\mathbf{x} = \mathbf{d}$ and solution ensues (U is upper triangular)
   c. Method: row reduction using elementary row operations (EROs); Gaussian elimination or Gauss-Jordan elimination
      i. Multiply a row (j) by a value (c)
      ii. Add a multiple (c) of one row (j) to another (k)
      iii. Interchange rows j and k

<u>Example Problems in Solving Systems of Linear Equations</u>

Prob. 1:

$$\begin{aligned} 3x_1 - x_2 + x_3 &= -1 \\ 9x_1 - 2x_2 + x_3 &= -9 \\ 3x_1 + x_2 - 2x_3 &= -9 \end{aligned}$$

Prob. 2:

$$\begin{aligned} 3x - y - 2z &= 0 \\ -6x + 2y + 6z &= 4 \\ 2x + y + 6z &= 13 \end{aligned}$$

Adapted from lecture outline by Prof. James Maneval, Fall 2019