ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Wednesday, Sept. 6

1. Method of least squares (application of normal equations)

   a. Given a data set: (x_i, y_i), $i = 1$ to M $\rightarrow$ data vectors **x** and **y**
   b. Define a set of weighted functions $\{f_j(x)\}_{j=1 \text{ to } N}$ that will hopefully fit the data:

   $$y(x) \approx \hat{y}(x) = \sum_{j=1}^{N} c_j f_j(x) \qquad \hat{y}(x) \text{ is the best fit curve}$$

   c. Coefficients $\{c_j\}_{j=1 \text{ to } N}$ found via $F^T F\mathbf{c} = F^T\mathbf{y}$ $\rightarrow$ $\mathbf{c} = \left(F^T F\right)^{-1} F^T\mathbf{y}$
   d. In Matlab: `c = F\y` (recommended)
   e. Could also use: `c = (F'F)\(F'y)` (academic interest only)

2. Back to the simple data set example: Applying the normal equation

   a. Recall that we found quadratic and linear fits to the following small data set:

i	x_i	y_i
1	1.0	1.1
2	2.0	3.2
3	4.0	5.2

   Quadratic fit: $y = c_0 + c_1 x + c_2 x^2$, where $\mathbf{c} = \begin{bmatrix} -1.7333 \\ 3.2000 \\ -0.3667 \end{bmatrix}$

   Linear fit: $y = d_0 + d_1 x$, where $\mathbf{d} = \begin{bmatrix} 0.1000 \\ 1.3143 \end{bmatrix}$

   b. Find the quadratic coefficients $\{c_j\}_{j=1 \text{ to } 3}$ using the normal equation as described above (not `c = F\y`).
      i. Form matrix F and data vector **y**.
      ii. Is the resulting coefficient vector **c** the same as before?
      iii. What do you notice about the matrix $F^T F$?
      iv. Compare to applying `c = F\y` in *Matlab*.

   c. Find the linear coefficients $\{d_j\}_{j=1 \text{ to } 2}$ using the normal equation.
      i. Form matrix F and data vector **y**.
      ii. Is the resulting coefficient vector **d** the same as before?
      iii. What do you notice about the matrix $F^T F$?
      iv. Compare to applying `d = F\y` in *Matlab*.

3. Next: Constrained least squares optimization (not in the textbook; supplemental reading to come)