ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Friday, Sept. 8

1. Possible limitations of normal equation:

   a. Sometimes produce large-magnitude and oscillatory weights if the basis functions overlap and are highly correlated (e.g., exponential functions)
   b. Problem to be solve might require that all weights be positive

2. Constrained least squares optimization (not in textbook):

   a. **Same as unconstrained LS:** Given a data set: (x_i, y_i), $i = 1$ to M $\rightarrow$ data vectors **x** and **y**
   b. **Same as unconstrained LS:** Define a set of weighted functions $\{f_j(x)\}_{j=1 \text{ to } N}$ that will hopefully fit the data:
   $$y(x) \approx \hat{y}(x) = \sum_{j=1}^{N} c_j f_j(x) \qquad \hat{y}(x) \text{ is the best fit curve}$$
   c. **Different:** Coefficients $\{c_j\}_{j=1 \text{ to } N}$ found via
   $$\left(F^T F\mathbf{c} + \gamma I\right) = F^T\mathbf{y} \quad \rightarrow \quad \mathbf{c} = \left(F^T F + \gamma I\right)^{-1} F^T\mathbf{y}\,,$$
   where γ is called a Lagrange multiplier
   d. In practice, start out with a very small value for γ and then increase it until the coefficients in **c** stop oscillating
   e. How it works: By adding a small value to the main diagonal of F^TF, its row vectors become less skew.
   f. 2-D analogy: $\mathbf{u}_1$ and $\mathbf{u}_2$ are basis vectors; small circle is solution they are trying to "reach" via the linear combination $c_1\mathbf{u}_1 + c_2\mathbf{u}_2$, where c_1 and c_2 are scalars. The coefficients c_1 and c_2 have to be large and have opposite algebraic signs if $\mathbf{u}_1$ and $\mathbf{u}_2$ are highly skewed (i.e., almost collinear).