

# MISCELLANEOUS FORMULAE

## Mensuration

Arc length:  $l = r\theta$   
 Area of sector:  $\Delta = \frac{1}{2}r^2\theta$   
 Volume of pyramid:  $V = \frac{1}{3}Ah$   
 Volume of cone:  $V = \frac{1}{3}\pi r^2h$   
 Volume of sphere:  $V = \frac{4}{3}\pi r^3$

In any triangle:  $a/\sin A = b/\sin B = c/\sin C$   
 In any triangle:  $a^2 = b^2 + c^2 - 2bc \cos A$   
 Area of triangle:  $\Delta = \frac{1}{2}bc \sin A$   
 Area of trapezium:  $\frac{1}{2}(a+b)h$

## Circular Functions

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$   
 $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$   
 $\tan(u \pm v) = (\tan u \pm \tan v)/(1 \mp \tan u \tan v)$   
 $\sin 2t = 2 \sin t \cos t$   
 $\cos 2t = \cos^2 t - \sin^2 t = 2 \cos^2 t - 1 = 1 - 2 \sin^2 t$   
 $\tan 2t = (2 \tan t)/(1 - \tan^2 t)$

$2 \cos u \cos v = \cos(u+v) + \cos(u-v)$   
 $2 \sin u \sin v = \cos(u-v) - \cos(u+v)$   
 $2 \sin u \cos v = \sin(u+v) + \sin(u-v)$   
 $\sin w + \sin z = 2 \sin \frac{1}{2}(w+z) \cos \frac{1}{2}(w-z)$   
 $\sin w - \sin z = 2 \cos \frac{1}{2}(w+z) \sin \frac{1}{2}(w-z)$   
 $\cos w + \cos z = 2 \cos \frac{1}{2}(w+z) \cos \frac{1}{2}(w-z)$   
 $\cos w - \cos z = -2 \sin \frac{1}{2}(w+z) \sin \frac{1}{2}(w-z)$

## Other Functions and Relations

Logarithms:  $\log_a x + \log_a y = \log_a xy$        $\log_a x - \log_a y = \log_a(x/y)$   
 $p \log_a x = \log_a x^p$        $\log_b x = \log_a x / \log_a b$

Arithmetic Series:  $a + (a+d) + \dots + [a + (n-1)d] = \frac{1}{2}n[2a + (n-1)d]$   
 Geometric Series:  $a + ar + \dots + ar^{n-1} = a(1-r^n)/(1-r)$ , ( $r \neq 1$ )  
 $a^n - x^n = (a-x)(a^{n-1} + a^{n-2}x + a^{n-3}x^2 + \dots + x^{n-1})$

Compound Interest:  $A_n = P(1+r/100)^n$   
 Binomial Series:  $a^n + \binom{n}{1}a^{n-1}x + \dots + \binom{n}{r}a^{n-r}x^r + \dots + x^n = (a+x)^n$

## Coordinate Geometry

Distance:  $d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$   
 Division of Interval:  $x = (mx_2 + nx_1)/(m+n)$ ,  $y = (my_2 + ny_1)/(m+n)$

Name	Equation of Conic	Foci	Equation of Directrices	Other Relationships
Ellipse	$x^2/a^2 + y^2/b^2 = 1$	$(\pm ae, 0)$	$x = \pm a/e$	$b^2 = a^2(1-e^2)$ , $0 < e < 1$
Hyperbola	$x^2/a^2 - y^2/b^2 = 1$	$(\pm ae, 0)$	$x = \pm a/e$	$b^2 = a^2(e^2 - 1)$ , $e > 1$
Parabola	$y^2 = 4ax$	$(a, 0)$	$x = -a$	$e = 1$

Polar equation of conic:  $l/r = 1 + e \cos \theta$

## Calculus

$f(x) = y$	$f'(x) = Df(x) = dy/dx$	$f(x) = y$	$f'(x) = Df(x) = dy/dx$
$x^n$	$nx^{n-1}$	$\arcsin(x/a)$	$1/\sqrt{a^2-x^2}$
$\log_e x = \ln x$	$x^{-1}$	$\arccos(x/a)$	$-1/\sqrt{a^2-x^2}$
$\log_e(ax+b)$	$a/(ax+b)$	$\arctan(x/a)$	$a/(a^2+x^2)$
$e^{kx}$	$ke^{kx}$	$g(x)h(x)$	$g'(x)h(x) + g(x)h'(x)$
$\sin kx$	$k \cos kx$	$g(x)/h(x)$	$\{g'(x)h(x) - g(x)h'(x)\}/\{h(x)\}^2$
$\cos kx$	$-k \sin kx$	$g \circ h(x) = g\{h(x)\}$	$g'\{h(x)\}h'(x)$
$\tan kx$	$k \sec^2 kx$		

## Matrices and Determinants

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$       If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $\det A = |A| = ad - bc$ .

## Probability and Statistics

Permutations:  ${}^n P_r = n(n-1) \dots (n-r+1)$   
 Combinations:  ${}^n C_r = \binom{n}{r} = \{n(n-1) \dots (n-r+1)\}/r!$   
 Probability:  $Pr(A) = 1 - Pr(A')$   
 $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$   
 $Pr(A \cap B) = Pr(A) \cdot Pr(B)$  if  $A, B$  are independent  
 $Pr(A|B) = Pr(A \cap B)/Pr(B)$  if  $Pr(B) \neq 0$

Sample mean:  $\bar{x} = (\Sigma x)/n$   
 Sample variance:  $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{\Sigma(x-a)^2 - \{\Sigma(x-a)\}^2/n}{n-1}$

Distribution	$Pr(X = x)$	Mean	Variance
Binomial	$\binom{n}{x} \pi^x (1-\pi)^{n-x}$	$n\pi$	$n\pi(1-\pi)$
Hypergeometric	$\binom{D}{x} \binom{N-D}{n-x} / \binom{N}{n}$	$\frac{n \cdot D}{N}$	$n \cdot \frac{D}{N} \left(1 - \frac{D}{N}\right) \frac{N-n}{N-1}$
Poisson	$e^{-\lambda} \cdot \lambda^x / x!$	$\lambda$	$\lambda$

Normal distribution curve:  $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$