PHYSICS 331: ADVANCED CLASSICAL MECHANICS

TEST 2 USEFUL RELATIONSHIPS

$$\vec{g} = -\vec{\nabla}\Phi \qquad d\Phi = -\frac{G\,dm}{r} \qquad \vec{F}_G = -\frac{GMm}{r^2}\hat{r}$$

$$\Phi_m = \oint_S \vec{g}\cdot d\vec{A} = -4\pi G\, m_{enclosed}$$

If $J(\alpha) \equiv \int_{x_1}^{x_2} f\left[y(\alpha,x), y'(\alpha,x); x\right] dx$, then $\frac{\partial J}{\partial \alpha} = 0$ implies

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad \text{Euler's equation.}$$

Second form of Euler's equation: $\frac{\partial f}{\partial x} - \frac{d}{dx}\left(f - y'\frac{\partial f}{\partial y'}\right) = 0$

Euler's equation with auxiliary conditions $\frac{\partial f}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_i'}\right) + \lambda\frac{\partial g}{\partial y_i} = 0$ & $g[y_i; x] = 0$

Lagrange's equation. $\frac{\partial L}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = 0$ where $L = T - U$