PHYSICS 331
Advanced Classical Mechanics
Useful Relationships

Hamilton's Equations:

If a system has generalised coordinates $\mathbf{q} = (q_1, q_2, ..., q_n)$, Lagrangian $\mathcal{L}$, and generalised momenta $p_i = \partial\mathcal{L}/\partial\dot{q}_i$, its Hamiltonian is defined as

$$\mathcal{H} = \sum_{i=1}^{n} p_i \, \dot{q}_i - \mathcal{L},$$

and Hamilton's equations are given by

$$\dot{q}_i = \frac{\partial\mathcal{H}}{\partial p_i} \qquad \text{and} \qquad \dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i}.$$

Inertial Forces:

$$\mathbf{F}_{Coriolis} = -2\, m\, \mathbf{\Omega} \times \dot{\mathbf{r}}$$

$$\mathbf{F}_{cfg} = -m\, \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$$

Central Forces:

For two objects m_1 and m_2 under the influence of a central force described by $F = \gamma/r^2$, the *effective* potential energy is given by

$$U_{eff}(r) = U(r) + \frac{\ell^2}{2\mu r^2}$$

where r describes the relative coordinate.

The transformed radial equation is given by

$$u''(\phi) = -\, u(\phi) \; - \; \frac{\mu}{\ell^2 u(\phi)^2}\; F$$

where $u = 1/r$, and has a solution given by

$$r(\phi) = \frac{c}{1 + \epsilon \cos\phi}$$

where $c = \ell^2/(\gamma\mu)$ and ϵ is related to the energy by

$$E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2 - 1).$$

For the case where $\epsilon < 1$, the trajectories are elliptical with semi-major and semi-minor axes given by:

$$a = \frac{\gamma}{2E} \qquad \text{and} \qquad b = \frac{\ell}{2\mu|E|}.$$