PHYSICS 331 ADVANCED CLASSICAL MECHANICS
Problem Set 32

Problem 1
Taylor: Chapter 10, Problem 39. HINT: Read through example 10.3 of Taylor.

Problem 2
Taylor: Chapter 10, Problem 42.

Problem 3
Taylor: Chapter 10, Problem 45.

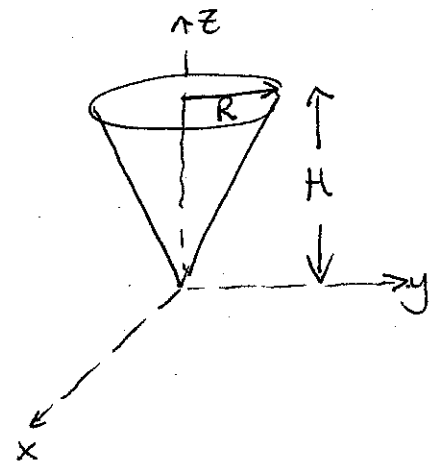
Problem 4
Taylor: Chapter 10, Problem 51.

Other Things
We will continue our journey with *Coupled Oscillations.* Read Marion and Thornton's chapter 12, sections 12.1 through 12.3. Please remember to submit your journal entry.

Taylor 10-39 Consider a top consisting of a uniform cone spinning freely about its tip at 1800 rpm.

H = height of top = 10cm

R = base radius = 2.5cm



To determine the angular frequency (Ω) at which it precesses, we note that $\Omega_{precess}$ of a top due to a weak torque is given by

$$\Omega_{precess} = \frac{Mgr}{\lambda_3 \omega}\,\hat{z}$$

where $L_z = \lambda_3 \omega$

and $\lambda_3 = \frac{3}{10}MR^2$ for a cone (see Taylor 10-59)

$\omega = 1800$ rpm $= 1800 \cdot \frac{2\pi}{60}$ rad/sec

Mgr = nett torque

$\Rightarrow r$ = distance from top's pivot to its centre-of-mass $= \frac{3}{4}H$ for a cone.

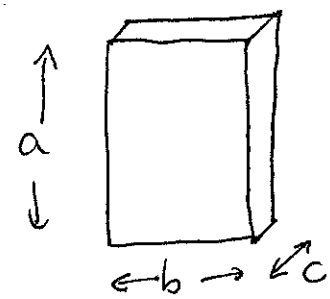
$$\therefore \quad \Omega_{press} = \frac{\cancel{M}g \cdot \left(\frac{3}{4}H\right)}{\frac{3}{10}\cancel{M}R^2 \cdot \omega} = \frac{\frac{\cancel{3}}{4} \cdot \frac{10}{\cancel{3}} \cdot g \cdot H}{R^2 \cdot \omega}$$

$$\therefore \quad \Omega_{precess} = \frac{\frac{5}{2} \cdot (9.8)(0.10)}{(0.025)^2 \cdot 1800 \cdot 2\pi/60} = 20.80 \text{ rad/sec}$$

$$\simeq 199 \text{ rpm.}$$

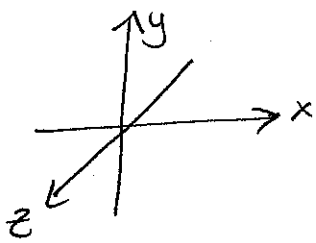
Taylor 10-42

Consider a book that is 30cm x 20cm x 3cm, held closed by a rubber band. It is then thrown up into the air spinning about an axis that is close to the book's shortest symmetry axis at 180 rpm.



$a = 30\text{cm}$
$b = 20\text{cm}$
$c = 3\text{cm}$

To determine the angular frequency of the small oscillations about its axis of rotation,



recall,
the moment tensor is given by

$$\underline{I} = \frac{m}{12}\begin{pmatrix} b^2+c^2 & & 0 \\ & a^2+c^2 & \\ 0 & & a^2+b^2 \end{pmatrix}$$

for rotation about the centre-of-mass

To determine the angular freq. about the short axis $\Rightarrow$ want Ω about ω_1

from (10-91), we showed $\Omega^2 = \dfrac{(\lambda_3-\lambda_2)(\lambda_3-\lambda_1)}{\lambda_1\lambda_2}\,\omega_3^2$

now,
$\lambda_1 = b^2+c^2$
$\lambda_2 = a^2+c^2$
$\lambda_3 = a^2+b^2$

$$\Rightarrow \Omega^2 = \frac{\left[(a^2+b^2)-(a^2+c^2)\right]\left[(a^2+b^2)-(b^2+c^2)\right]}{(b^2+c^2)(a^2+c^2)}\,\omega_3^2$$

$$= \frac{(b^2-c^2)(a^2-c^2)}{(b^2+c^2)(a^2+c^2)}\,\omega_3^2$$

$$\therefore \Omega^2 = \frac{(20^2-3^2)(30^2-3^2)}{(20^2+3^2)(30^2+3^2)}\,(180\text{rpm})^2$$

Taylor 10-42 (cont'd)

$$\Omega = \sqrt{0.937}\,\omega_3 = 0.968\,\omega_3 = 0.968\,(180 \text{ rpm}) = 174 \text{ rpm}.$$

If spinning about an axis close to the longest axis, we follow the same procedure as above except we interchange $1 \leftrightarrow 3$ for rotation about a dim^{n}

$$\therefore \quad \Omega^2 = \frac{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}{\lambda_3 \lambda_2}\,\omega_3^2$$

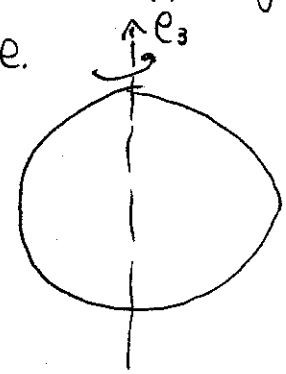
$$= \frac{(c^2 - a^2)(b^2 - a^2)}{(c^2 + a^2)(b^2 + a^2)}\,\omega_3^2$$

$$\therefore \quad \Omega = \sqrt{0.377}\,\omega_3 = 0.614\,\omega_3 = 110.5 \simeq 111 \text{ rpm}.$$

Taylor 10-45

Because of the Earth's equatorial bulge, its moment about the polar axis is slightly greater than the other two moments

i.e.



$\lambda_1 = \lambda_2$, but $\lambda_3 \simeq 1.00327\,\lambda_1$

(a) The free precession of this body (the Earth) is given by

$$\Omega_b = \frac{\lambda_1 - \lambda_3}{\lambda_1}\,\omega_3$$

$\hat{e}_3$ is the polar axis

$\Rightarrow \omega_3 = 1$ rev/day $= 2\pi$ rad/day.

$$\therefore \Omega_b = \left|\frac{\lambda_1 - 1.00327\lambda_1}{\lambda_1}\right| \omega_3$$

$$\Omega_b = (0.00327)\cdot\omega_3 = \frac{2\pi}{T_b}$$

$$\therefore T_b = \frac{2\pi}{\Omega_b} = \frac{2\pi}{0.00327\cdot\omega_3} = \frac{\cancel{2\pi}}{0.00327\cdot\cancel{2\pi}/\text{day}}$$

$$= \frac{1}{(0.00327)} \text{ days}$$

$\therefore T_b = 305.8$ days $\simeq 306$ days, as required

(b) The angle b/w $\vec{\omega}$ & the polar axis $= 0.2$ arc.sec $= 0.2\cdot\left(\frac{1}{60}\times\frac{1}{60}\right)$ degrees.

$\alpha \simeq 9.7\times10^{-7}$ rad

$\sin\alpha \approx \alpha$

$$\therefore \Omega_s = \frac{\omega\sqrt{\lambda_3^2 + (\lambda_1^2 - \lambda_3^2)\sin^2\alpha}}{\lambda_1} \simeq \frac{\omega}{\lambda_1}\sqrt{\lambda_3^2 + \sim 0}$$

$$= \frac{\lambda_3\omega}{\lambda_1}$$

$\approx 1\omega \Rightarrow 1$ rev/day.