Consider "majority" example again:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
<th>( \overline{F} )</th>
<th>&quot;Maxterms&quot;</th>
<th>&quot;Minterms&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( M_0 = A + B + C )</td>
<td>( m_0 = A\overline{B}\overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( M_1 = A + B + \overline{C} )</td>
<td>( m_1 = A\overline{B}C )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( M_2 = A + \overline{B} + C )</td>
<td>( m_2 = A\overline{B}C )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( M_3 = A + \overline{B} + \overline{C} )</td>
<td>( m_3 = \overline{A}BC )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( M_4 = \overline{A} + B + C )</td>
<td>( m_4 = \overline{A}\overline{B}C )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( M_5 = \overline{A} + B + \overline{C} )</td>
<td>( m_5 = \overline{A}\overline{B}C )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( M_6 = \overline{A} + \overline{B} + C )</td>
<td>( m_6 = \overline{A}BC )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( M_7 = A + \overline{B} + \overline{C} )</td>
<td>( m_7 = ABC )</td>
</tr>
</tbody>
</table>

**Sum of products for \( \overline{F} \):**

\[
\overline{F} = m_0 + m_1 + m_2 + m_4 \\
= \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + A\overline{B}\overline{C}
\]

**Take complement to get \( F \):**

\[
F = \overline{\overline{F}} = \overline{\overline{A}\overline{B}\overline{C}} + \overline{\overline{A}\overline{B}C} + \overline{\overline{A}BC} + \overline{A\overline{B}\overline{C}} \\
= (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + C) \\
= M_0 \cdot M_1 \cdot M_2 \cdot M_4
\]

"Product of sums" or "Product of maxterms"

**Note:**

- Each "factor" corresponds to a 0 in truth table for \( F \)
- Each factor equals 0 only for one input combination
- Since factors are ANDed, any one of them equal to 0 makes the result equal to 0; else result is 1.
- Convention for complementing in maxterms is opposite to minterms.
- \( M_j = \overline{m_j} \) [Corresponding maxterms + minterms are complements]
Please read in Section 11.5 - analogous to our development of sum-of-products:

- 2-level implementation with OR-AND:

- 2-level implementation with NOR only:

NOR is a universal gate: can implement AND, OR, NOT with NOR only, so can implement any Boolean function with NOR.

- Can use Boolean algebra to expand simplified expression into standard product of max terms form (that is, include missing terms).

**Example:**

F = \( A \overline{C} + (A + \overline{B}) \overline{C} \)

Recall distributive property: \( X +YZ = (X+Y)(X+Z) \)

\[
F = \left( \frac{A \overline{C} + A + \overline{B}}{A + \overline{C}} \right) \left( \frac{A \overline{C} + C}{\overline{A} + C} \right) \\
= \frac{(A + \overline{B} + \overline{C}) \cdot (\overline{A} + C)}{\overline{A} + C} = (A + \overline{B} + \overline{C})(\overline{A} + C + B \overline{B}) \\
= (A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + C) = M_3 M_4 M_6 \]
Can also use K-maps to simplify product-of-sums expressions:

- Place the 0's in the K-map
- Circle groups of 0's as before
- Write the minterms that characterize each group of zeros
- Take product of minterms

**Ex:** Majority

<table>
<thead>
<tr>
<th></th>
<th>BC</th>
<th>00</th>
<th>01</th>
<th>11</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F = \]