

LAPLACE TRANSFORM INTRODUCTION

Consider time signals  $x(t)$  that equal zero for  $t < 0$ .

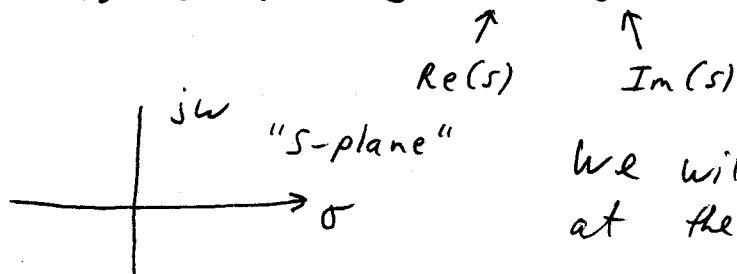
Why? Because the Laplace transform helps us to understand and design systems, and systems are "turned on" at some time.

(Fourier transform is primarily for signal analysis and design.)

"One-sided" Laplace transform of  $x(t)$ :

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt$$

What is  $s$ ?  $s = \sigma + j\omega$  is a complex number.



We will often look at the "s-plane".

Notation:  ~~$x(t)$~~   $x(t) \leftrightarrow X(s)$

$$X(s) = \mathcal{L}\{x(t)\}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

(We'll do inverse Laplace transform next class.)

Recall Fourier transform:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

= Laplace transform along imaginary axis  $s = j\omega$ .

Signals in table multiplied by  $u(t)$

Compute some Laplace transforms:

$$\mathcal{L}\{s(t)\} =$$

$$\mathcal{L}\{u(t)\} =$$

"Region of convergence" (Roc) =

$$\mathcal{L}\{e^{-2t} u(t)\} =$$

R.o.c. =

See tables inside back cover of text for more Laplace transform pairs.

We also have Laplace transform properties to extend the table to other signals.

Other transforms to compute:

$$\mathcal{L}\{\cos 10t \cdot u(t)\} =$$

$$\mathcal{L}\left\{ \begin{array}{c} 2 \\ \hline \text{---} \\ \hline 0 \end{array} \begin{array}{c} \text{---} \\ \hline 3 \end{array} \rightarrow t \right\} =$$

$$\mathcal{L}\left\{ \begin{array}{c} \text{ramp} \\ \text{---} \\ \hline 0 \end{array} \rightarrow t \right\} =$$