ELEC 320
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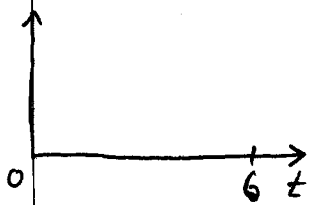
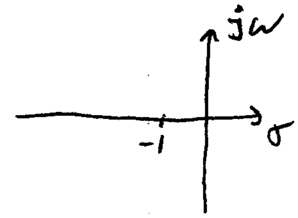
LAPLACE TRANSFORM AND INVERSE L.T.

Relation between time signals and s-plane:

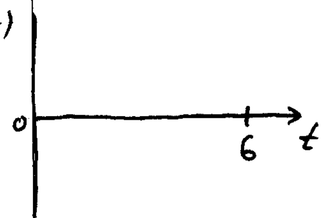
Time signal	Laplace tr.	Pole(s)	s-plane

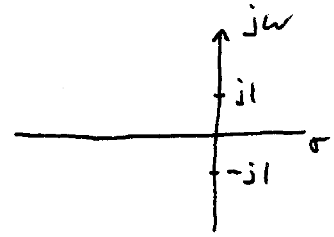
$e^{-t}u(t)$



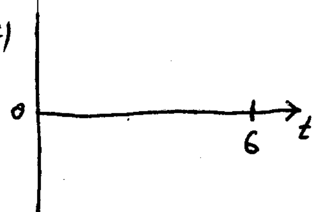


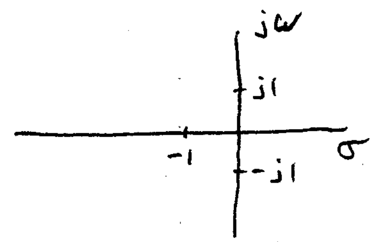
$\cos t \cdot u(t)$



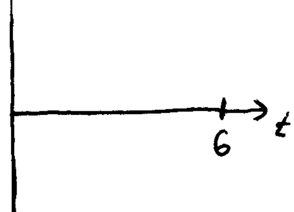


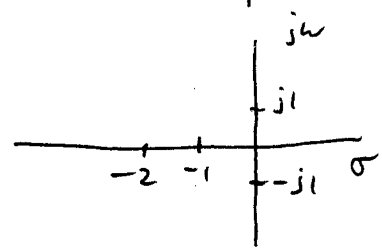
$e^{-t}\cos t\, u(t)$



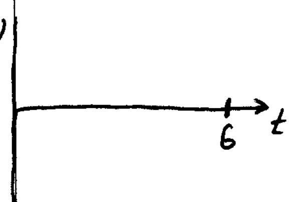


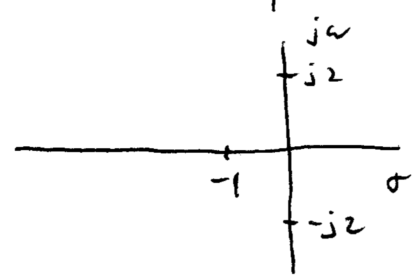
$e^{-2t}\cos t\, u(t)$





$e^{-t}\cos 2t\, u(t)$





What are the effects of:

① moving a pole <u>left</u> along σ-axis?

② moving a pole further out along jω-axis?

Motivation for "partial fraction" expansion to perform inverse Laplace transform:

$$\mathcal{L}\{3e^{-2t}u(t)\} =$$

$$\mathcal{L}\{[4e^{-t} + e^{-10t}]u(t)\} =$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1} + \frac{9}{s+10}\right\} =$$

$$\mathcal{L}^{-1}\left\{\frac{9}{(s+1)(s+10)}\right\}$$

Inverse Laplace transform examples:

(1) $\dfrac{s+17}{s^2+4s-5}$ (2) $\dfrac{3s-5}{s^3+3s^2+7s+5}$

(3) $\dfrac{16s+43}{s^3+4s^2-3s-18}$ (4) $\dfrac{s^3+2s-4}{s^2+4s-2}$

(5) $\dfrac{1}{s+2} + \dfrac{s+17}{s^2+4s-5}e^{-2\cdot s}$

(We'll do in class: analytically & with MATLAB)

<u>Add to your Laplace transform table:</u>

For a complex ~~pole~~ pole $p_1 = \sigma + j\omega$: ($*$ = complex conjugate)

(A) $$\frac{c_1}{s-p_1} + \frac{c_1^*}{s-p_1^*} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} 2|c_1|e^{\sigma t}\cos(\omega t + \angle c_1)\,u(t)$$

(B) $$\frac{b_1 s + b_0}{(s-\sigma)^2+\omega^2} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} e^{\sigma t}\left[b_1\cos\omega t + \frac{b_0+\sigma b_1}{\omega}\sin\omega t\right]u(t)$$

$$= \sqrt{b_1^2 + \left(\frac{b_0+\sigma b_1}{\omega}\right)^2}\, e^{\sigma t}\cos\left[\omega t + \tan^{-1}\frac{(-b_0-\sigma b_1)/\omega}{b_1}\right]u(t)$$