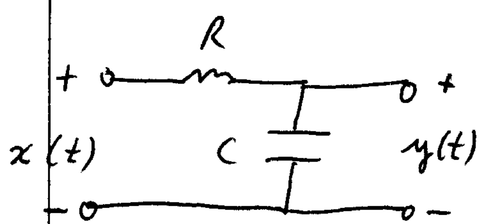
ELEC 320
R. Kozick



## Laplace Transform Applied to Linear Systems




$x(t) = 0$ for $t < 0$

$y(0^-) =$ Initial capacitor voltage

Recall that $\frac{dy}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t)$

Laplace transform of differential equation:

Exercise: Find $y(t)$ if $RC = 10^{-3}$, $y(0) = -1V$, $x(t) = 2u(t)$.

---

Let us focus on the "zero state response" (ZSR) = output due to input signal with zero "initial conditions".

Circuits can be drawn in the s-domain:

$$R \rightarrow R, \quad C \rightarrow \frac{1}{sC}, \quad L \rightarrow sL$$

[See pages 400-406 of the Lathi text for derivation and how non-zero initial conditions can be included]


$$Y(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} X(s)$$

$$= \frac{X(s)}{1 + (RC)s} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot X(s)$$

(Can also see this from differential eq.)

Important Definition: "Transfer function"

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$$

$$= \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \quad \text{for } RC \text{ circuit.}$$

$\therefore \quad Y(s) = H(s) \cdot X(s) \qquad$ [ for ZSR ]

Laplace transform property: Multiplication in s-domain $\longleftrightarrow$ Convolution in time-domain.

$$y(t) = \boxed{h(t)} * x(t)$$

What must h(t) be? ________________

$\therefore$ Transfer function $H(s) = \mathcal{L}\{ \qquad \qquad \}$

What is the impulse response of RC circuit?

** NOTE: Impulse response $h(t) \neq \frac{y(t)}{x(t)}$.

But in s-domain: $H(s) = \frac{Y(s)}{X(s)}$

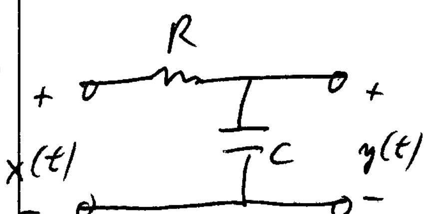
Exercises: Find the transfer function $H(s)$ of the following linear, time-invariant systems.

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

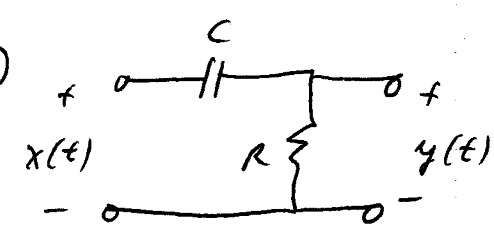
1. Ideal integrator
$$y(t) = \int_0^t x(\lambda)\, d\lambda$$

2. Differentiator
$$y(t) = \frac{dx}{dt}$$

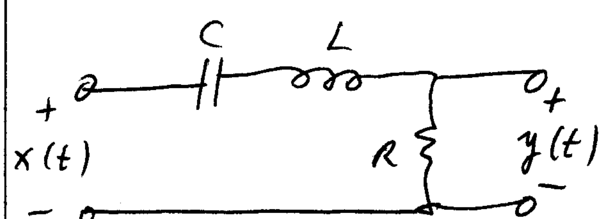
3.




4.




5.



Usually transfer function $H(s) = \dfrac{\text{Numerator polynomial}}{\text{Denom. polynomial}}$

"ZEROS" of system = roots of numerator of $H(s)$

"POLES" of system = roots of denominator of $H(s)$

Exercise: Plot the poles and zeros in the s-plane for the 5 systems on page 3.

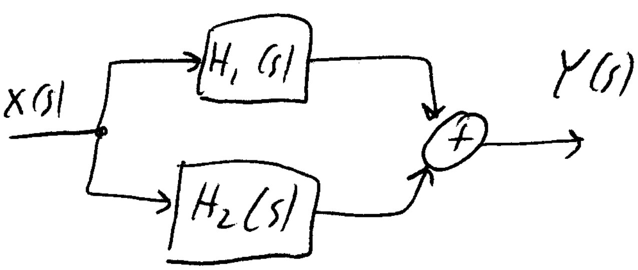
---

Connecting systems together: (See Section 7.6 in text)

Series:

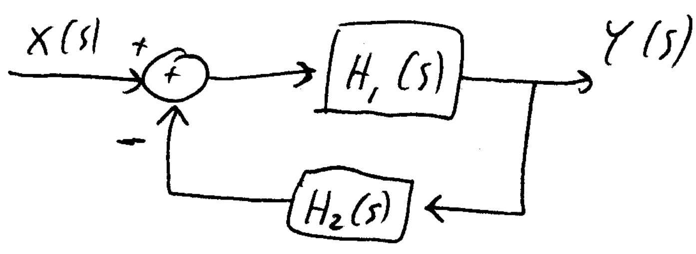

$H(s) = \dfrac{Y(s)}{X(s)} =$

Parallel:




$H(s) = \dfrac{Y(s)}{X(s)} =$

Negative Feedback:




used for amplifiers & control systems

$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 + H_1(s) \cdot H_2(s)}$$