

Table 6.1

A Short Table of (Unilateral) Laplace Transforms

$f(t)$	$F(s)$
1 $\delta(t)$	1
2 $u(t)$	$\frac{1}{s}$
3 $tu(t)$	$\frac{1}{s^2}$
4 $t^n u(t)$	$\frac{n!}{s^{n+1}}$
5 $e^{\lambda t} u(t)$	$\frac{1}{s-\lambda}$
6 $te^{\lambda t} u(t)$	$\frac{1}{(s-\lambda)^2}$
7 $t^n e^{\lambda t} u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a $\cos bt u(t)$	$\frac{s}{s^2 + b^2}$
8b $\sin bt u(t)$	$\frac{b}{s^2 + b^2}$
9a $e^{-at} \cos bt u(t)$	$\frac{s+a}{(s+a)^2 + b^2}$
9b $e^{-at} \sin bt u(t)$	$\frac{b}{(s+a)^2 + b^2}$
10a $re^{-at} \cos(bt + \theta) u(t)$	$\frac{(r \cos \theta)s + (ar \cos \theta - br \sin \theta)}{s^2 + 2as + (a^2 + b^2)}$
10b $re^{-at} \cos(bt + \theta) u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb}$
10c $re^{-at} \cos(bt + \theta) u(t)$	$\frac{As+B}{s^2 + 2as + c}$
	$r = \sqrt{\frac{A^2 c + B^2 - 2ABa}{c-a^2}}, \theta = \tan^{-1} \frac{As-B}{A\sqrt{c-a^2}}$
	$b = \sqrt{c-a^2}$
10d $e^{-at} \left[A \cos bt + \frac{B-Aa}{b} \sin bt \right] u(t)$	$\frac{As+B}{s^2 + 2as + c}$
	$b = \sqrt{c-a^2}$

Table 6.2

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^t f(\tau) d\tau$	$\frac{1}{s} F(s)$
	$\int_{-\infty}^t f(\tau) d\tau$	$\frac{1}{s} F(s) + \frac{1}{s} \int_{-\infty}^{0^-} f(t) dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0 t}$	$F(s-s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^{\infty} F(z) dz$
Scaling	$f(at), a \geq 0$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j} F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} sF(s)$ (poles of $sF(s)$ in LHP)