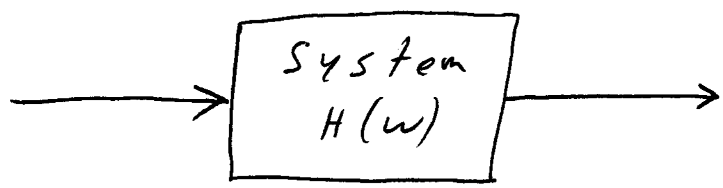
Frequency Response

The frequency response $H(\omega)$ of a system is a complex number for each frequency ω (rad/sec).

Polar form $\Rightarrow$ $H(\omega) = |H(\omega)| \cdot \angle H(\omega)$

What does $H(\omega)$ tell us about a system?

Consider applying an input signal to the system that is a cosine with frequency ω_0:

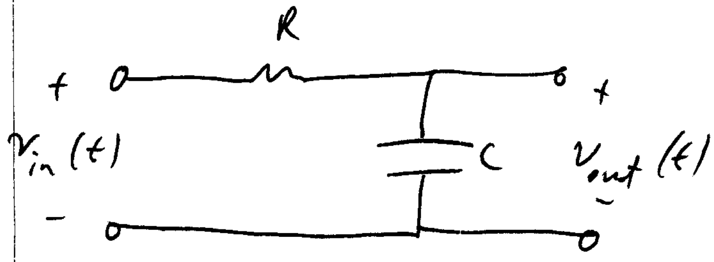


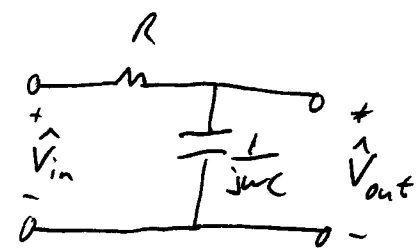
Time Signal: $A \cos[\omega_0 t]$ $\qquad$ ____ $\cos[\omega_0 t +$ ____ $]$

Phasor: $A \angle 0°$

$\therefore$ $H(\omega)$ describes the "amplitude gain" and the "phase shift" of the system at each frequency ω.

Example: Consider the RC circuit that we demonstrated in class.





$$\hat{V}_{out} = \frac{1/j\omega C}{R + \frac{1}{j\omega C}} \hat{V}_{in} = \left(\frac{1}{1 + j\omega RC} \right) \hat{V}_{in}$$

$$H(\omega) = \frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{1}{1 + j\omega RC}$$

For the values we considered in class:

$R = 1.1 \text{ k}\Omega$

$C = 0.22 \ \mu F$

$\omega = 2\pi \cdot 658$ rad/sec.

$$H(2\pi\, 658) = \frac{1}{1 + j1} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$= 0.707 \angle -45^\circ$$

$v_{in}(t) = 0.1 \cos(2\pi 658 t)$ | $\hat{V}_{in} = 0.1 \angle 0^\circ$

$v_{out}(t) = 0.0707 \cos(2\pi 658 t - 45^\circ)$ | $\hat{V}_{out} = 0.0707 \angle -45^\circ$