

Homework 13

Date Assigned: Wednesday, October 14, 1998
Date Due: Friday, October 16, 1998

Reading: Sections 4.1 and 4.2 in the text on Fourier Series.

1. Please experiment with the Simulink demonstration of Fourier series that we used in class. You can access the demonstration from the course home page under Class Notes and Demonstrations.

You do not have to submit anything for this part.

2. In the Simulink demonstration fsgen.mdl, the three individual sine waves that are added together are

\[ f_1(t) = \sin(2\pi t) \]
\[ f_3(t) = \sin(2\pi 3t) \]
\[ f_5(t) = \sin(2\pi 5t) \]

Compute the “inner products” of pairs of these sine waves over the interval \([0, 1]\). That is, compute the following integrals:

\[
\int_0^1 f_1(t) f_1(t) \, dt, \quad \int_0^1 f_1(t) f_3(t) \, dt, \quad \int_0^1 f_1(t) f_5(t) \, dt \\
\int_0^1 f_3(t) f_3(t) \, dt, \quad \int_0^1 f_3(t) f_5(t) \, dt, \quad \int_0^1 f_5(t) f_5(t) \, dt
\]

Some useful identities:

\[ \sin^2 a = \frac{1}{2} [1 - \cos(2a)] , \quad \sin a \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)] \]

You may find it useful to sketch the integrand in each case.

3. What does it mean geometrically when two vectors have an inner product of zero? Draw a picture of this situation in two dimensions.