DISCRETE FOURIER TRANSFORM (DFT)

Continuous-time Fourier transform:

$$G(f) = F[g(t)] = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$g(t) = F^{-1}[G(f)] = \int_{-\infty}^{\infty} G(f)\, e^{+j2\pi f t}\, df$$

Suppose we have N samples $g_0, g_1, \ldots, g_{N-1}$

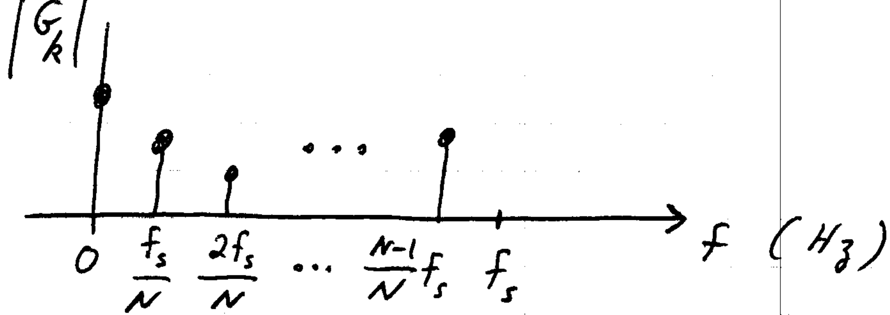
$$g_n = g(n T_s), \quad n = 0, 1, \ldots, N-1$$

T_s = sample spacing, $f_s = \frac{1}{T_s}$ = sampling frequency.

DFT: $$G_k = \sum_{n=0}^{N-1} g_n\, e^{-j \frac{2\pi}{N} kn}, \quad k = 0, 1, \ldots, N-1$$

IDFT: $$g_n = \frac{1}{N} \sum_{k=0}^{N-1} G_k\, e^{+j \frac{2\pi}{N} kn}, \quad n = 0, 1, \ldots, N-1$$

The $G_0, G_1, \ldots, G_{N-1}$ are samples in the frequency spectrum:



Spaced by $\frac{f_s}{N}$

For real g_n, the DFT is unique only up to $\frac{1}{2} f_s$.

To compute the DFT:

$$G_0 = g_0 \cdot 1 + g_1 \cdot 1 + \dots + g_{N-1} \cdot 1$$

$$G_1 = g_0 \cdot 1 + g_1 \cdot e^{-j\frac{2\pi}{N}} + \dots + g_{N-1} e^{-j\frac{2\pi}{N}(N-1)}$$

$$\vdots$$

$$G_{N-1} = g_0 \cdot 1 + g_1 \cdot e^{-j\frac{2\pi}{N} 1 \cdot (N-1)} + \dots + g_{N-1} e^{-j\frac{2\pi}{N}(N-1)^2}$$

How many "multiply and add" operations?

Fast Fourier Transform (FFT):

A more efficient algorithm to compute the DFT that requires $\simeq N \cdot \log_2 N$ multiply/adds.

Table 2.3 Savings in complex multiplications and additions when the FFT is used instead of the DFT.

N	DFT: *Number of complex multiplications*	DFT: *Number of complex additions*	FFT: *Number of complex multiplications*	FFT: *Number of complex additions*	*Ratio of DFT multiplications to FFT multiplications*	*Ratio of DFT additions to FFT additions*
2	4	2	1	2	4	1
4	16	12	4	8	4	1.5
8	64	56	12	24	5.3	2.3
16	256	240	32	64	8.0	3.75
32	1 024	992	80	160	12.8	6.2
64	4 096	4 032	192	384	21.3	10.5
128	16 384	16 256	448	896	36.6	18.1
256	65 536	65 280	1 024	2 048	64.0	31.9
512	262 144	261 632	2 304	4 608	113.8	56.8
1024	1 048 576	1 047 552	5 120	10 240	204.8	102.3
2048	4 194 304	4 192 256	11 264	22 528	372.4	186.1
4096	16 777 216	16 773 120	24 576	49 152	682.7	341.3
8192	67 108 864	67 100 672	53 248	106 496	1 260.3	630.0