ELEC 470
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# Single Sideband (SSB)

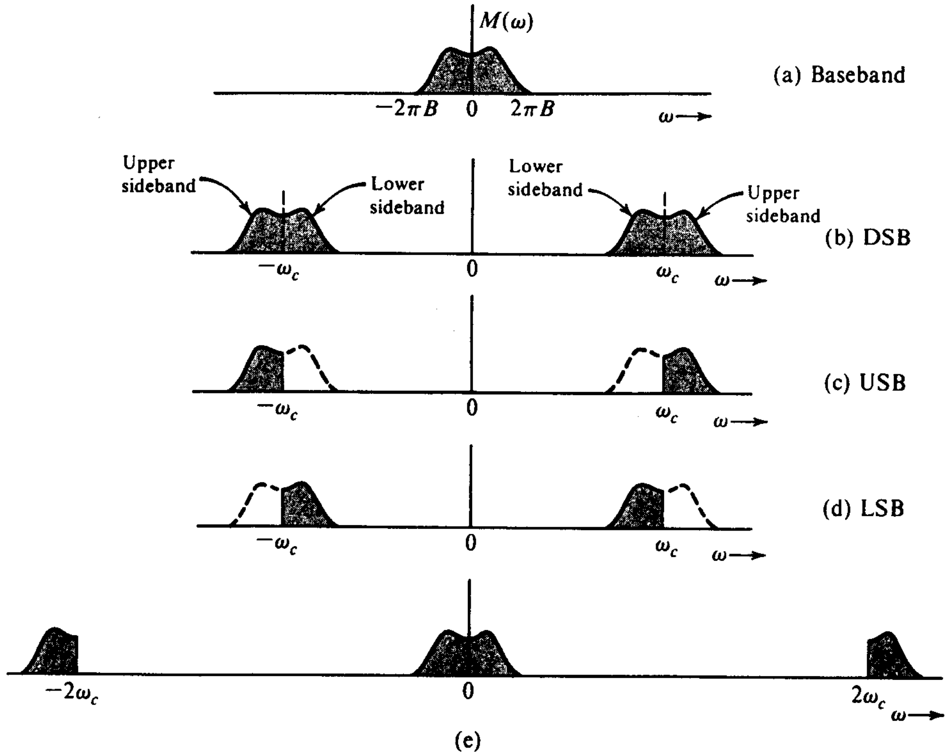
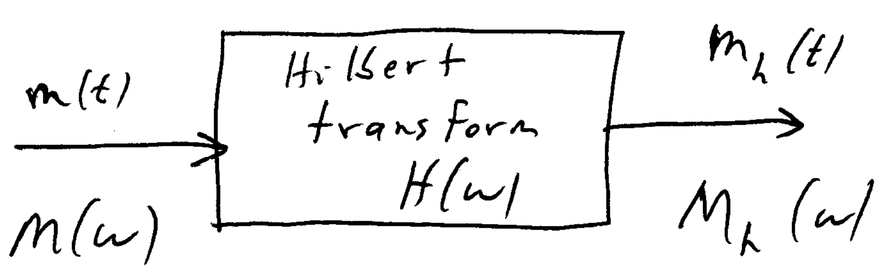



**Figure 4.15** SSB spectra.

Recover message from SSB with coherent demodulator.

Hilbert transform: Apply $-\frac{\pi}{2}$ phase shift to <u>all</u> frequencies.



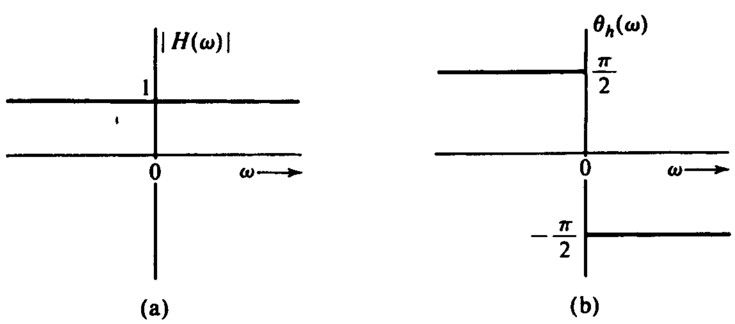



**Figure 4.17** Transfer function of an ideal $\pi/2$ phase shifter (Hilbert transformer).

is the Hilbert transform of $m(t)$. From Eq. (4.14b), it follows that if $m(t)$ is passed through a transfer function $H(\omega) = -j \text{ sgn }(\omega)$, then the output is $m_h(t)$, the Hilbert transform of $m(t)$. Because

$$H(\omega) = -j \text{ sgn }(\omega) \tag{4.16a}$$

$$= \begin{cases} -j = 1e^{-j\pi/2} & \omega > 0 \\ j = 1e^{j\pi/2} & \omega < 0 \end{cases} \tag{4.16b}$$

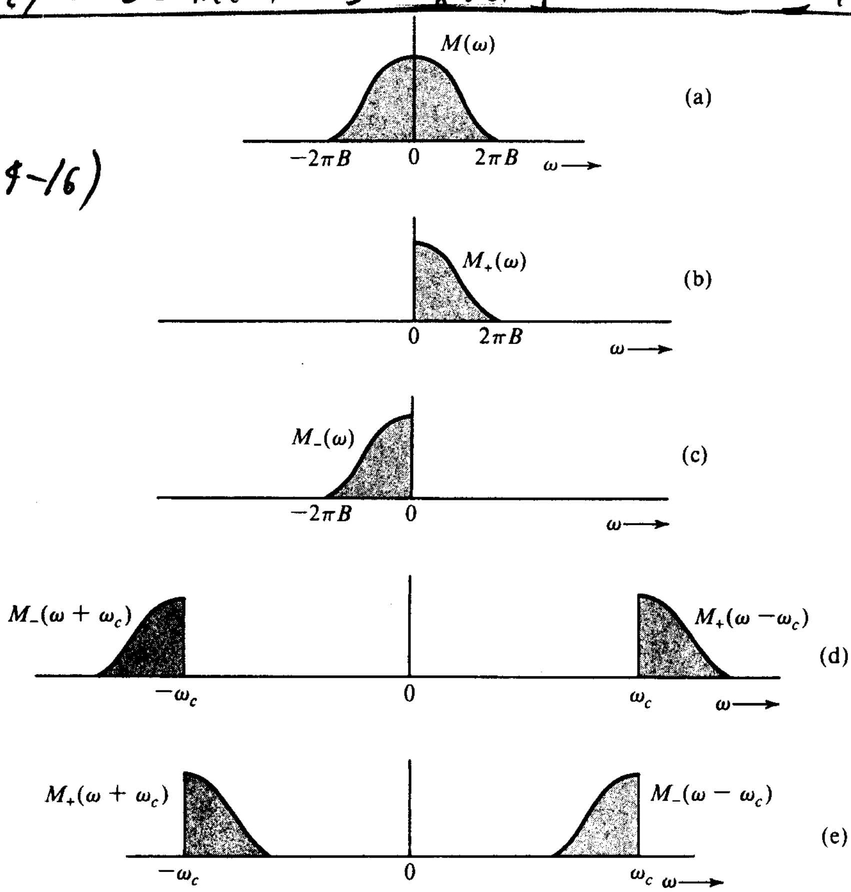
it follows that $|H(\omega)| = 1$ and that $\theta_h(\omega) = -\pi/2$ for $\omega > 0$ and $\pi/2$ for $\omega < 0$, as shown in Fig. 4.17. Thus, if we delay the phase of every component of $m(t)$ by $\pi/2$ (without changing its amplitude), the resulting signal is $m_h(t)$, the Hilbert transform of $m(t)$. Therefore, a Hilbert transformer is an ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$.

What do the spectra of the following Complex signals look like?

$m_+(t) = \frac{1}{2}[m(t) + j\,m_h(t)] \leftrightarrow M_+(\omega) = ?$

$m_-(t) = \frac{1}{2}[m(t) - j\,m_h(t)] \leftrightarrow M_-(\omega) = ?$

(Fig. 4-16)



Use Hilbert transform to generate SSB:

We can now express the SSB signal in terms of $m(t)$ and $m_h(t)$. From Fig. 4.16d it is clear that the USB spectrum $\Phi_{USB}(\omega)$ can be expressed as

$$\Phi_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

The inverse transform of this equation yields

$$\varphi_{USB}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}$$

Substituting Eqs. (4.13) in the preceding equation yields

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t \tag{4.17a}$$

Using a similar argument, we can show that

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t \tag{4.17b}$$

Hence, a general SSB signal $\varphi_{SSB}(t)$ can be expressed as

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t \tag{4.17c}$$

where the minus sign applies to USB and the plus sign applies to LSB.

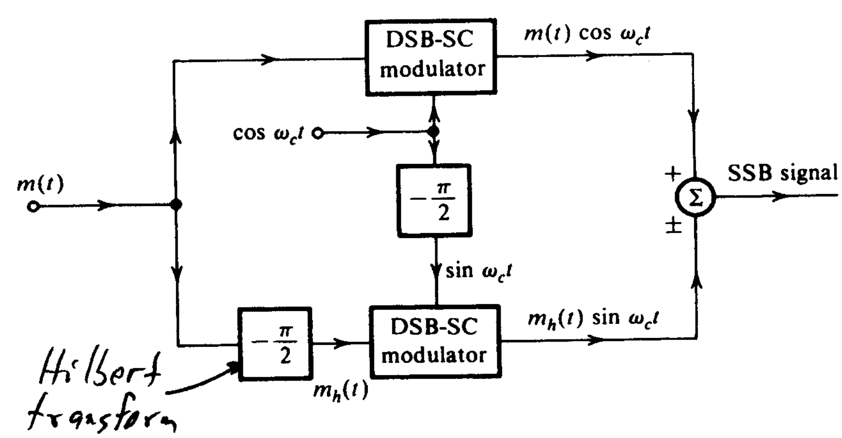



**Figure 4.20** SSB generation by phase-shift method.

Note similarity to QAM!

Impulse response of Hilbert transform filter:

$$h_h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \frac{1}{\pi t}$$

$$m_h(t) = m(t) * h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha}\, d\alpha$$

Example: $m(t) = \cos(10t)$, $\omega_c = 100$ rad/sec

# Vestigial Sideband (VSB):

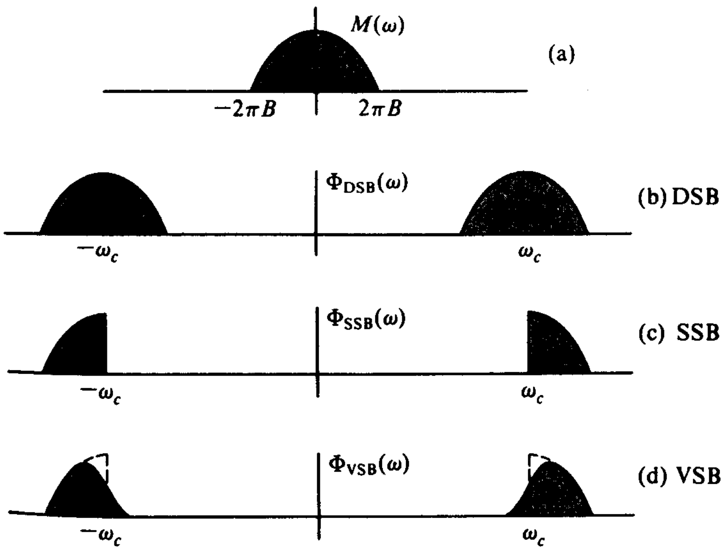



**Figure 4.21** Spectra of the modulating signal and corresponding DSB, SSB, and VSB signals.

## Example: