Class example on independent random variables

Problem Solution : Yates and Goodman,3.7.1

Problem 3.7.1

Flip a fair coin 100 times and let X be the number of heads in the first 75 flips and Y be the number of heads in the last 25 flips. We know that X and Y are independent and can find their PMFs easily.

$$P_X(x) = \begin{cases} \binom{75}{x}(1/2)^{75} & x = 0, 1, \ldots, 75 \\ 0 & \text{otherwise} \end{cases} \qquad P_Y(y) = \begin{cases} \binom{25}{y}(1/2)^{25} & y = 0, 1, \ldots, 25 \\ 0 & \text{otherwise} \end{cases}$$

The joint PMF of X and N can be expressed as the product of the marginal PMFs because we know that X and Y are independent.

$$P_{X,Y}(x,y) = \begin{cases} \binom{75}{x}\binom{25}{y}(1/2)^{100} & x = 0, 1, \ldots, 75 \ \ y = 0, 1, \ldots, 25 \\ 0 & \text{otherwise} \end{cases}$$