

Correlation Functions of a Homogeneously Driven Granular Fluid in Steady State

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Model & Simulation

- ▶ Hard Spheres, 3 dim.

- ▶ Dissipation

$$\vec{n} \cdot (\vec{v}_1' - \vec{v}_2') = -\epsilon \vec{n} \cdot (\vec{v}_1 - \vec{v}_2)$$

ϵ = coefficient of normal restitution

- ▶ Nonequilibrium Steady State

- ▶ Volume Driving

- ▶ $\frac{d}{dt} \vec{v}_i = \left(\frac{d}{dt} \vec{v}_i \right)_{\text{coll}} + \vec{\xi}_i(t)$ [van Noije et al. 1999]

- ▶ $\xi_i(t)$ Gaussian white noise with

$$\langle \vec{\xi} = 0 \rangle \text{ and } \langle \xi_{i\alpha}(t) \xi_{j\beta}(t') \rangle = \xi_0^2 \delta_{ij} \delta_{\alpha\beta} \delta(t - t')$$

- ▶ to conserve total momentum globally fixed pairs with opposite kicks

- ▶ Event Driven Simulations

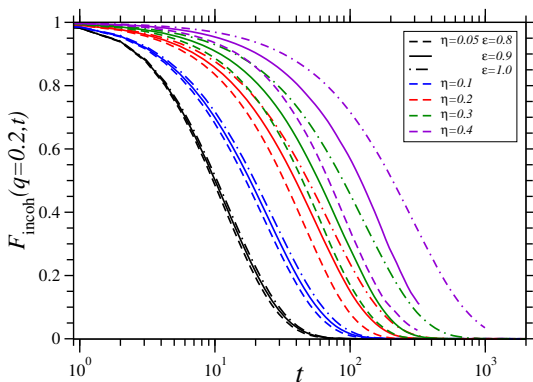
- ▶ $N = 200000$

- ▶ Volume Fractions $\eta = 0.05, 0.1, 0.2, 0.3, 0.4$

- ▶ $\epsilon = 1.0$ (elastic), 0.9, 0.8

Incoherent Intermediate Scattering Function

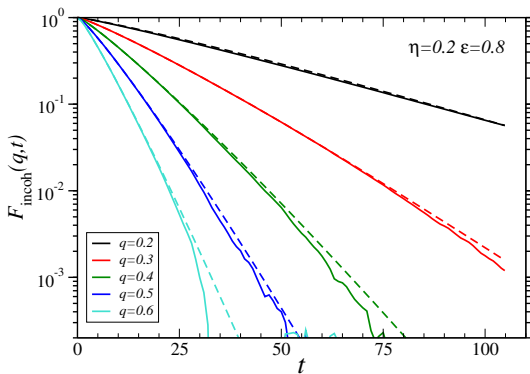
$$F_{\text{incoh}}(q, t) = \left\langle \frac{1}{N} \sum_{i=1}^N e^{i\vec{q} \cdot (\vec{r}_i(t) - \vec{r}_i(0))} \right\rangle$$



- ▶ dependence on ϵ & η
- ▶ not dense enough for glassy behavior
- ▶ Gaussian approximation?
- ▶ relaxation time τ

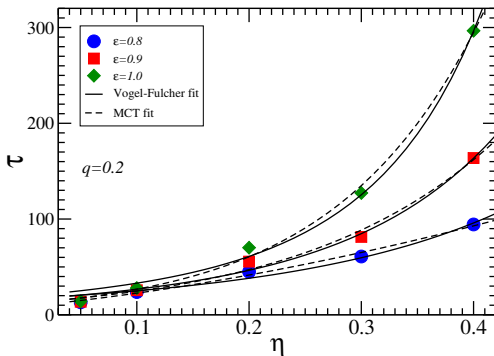
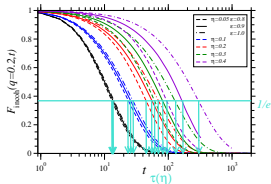
Gaussian Approximation of $F_{\text{incoh}}(q, t)$

$$F_{\text{incoh}}(q, t) = e^{-\frac{1}{6} q^2 \langle \frac{1}{N} \sum_{i=1}^N (\vec{r}_i(t) - \vec{r}_i(0))^2 \rangle}$$



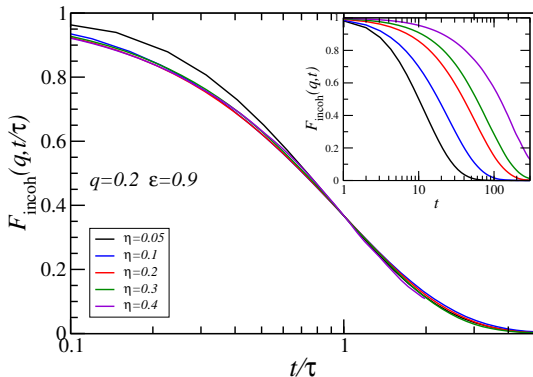
- ▶ good approximation
- ▶ similarly for other η, ϵ, q

Relaxation Time



- ▶ τ rapidly increasing with increasing η
- ▶ faster increase for larger ϵ
- ▶ compare: $\eta_{\text{glass}} = 0.58$

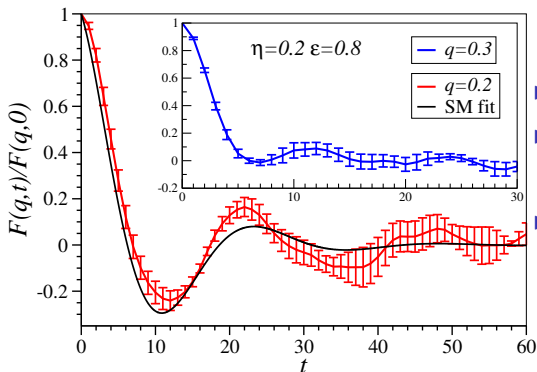
Incoherent Intermediate Scattering Function



- ▶ time superposition for all η but smallest
- ▶ curves are of same shape
- ▶ similarly for other q, ϵ

Intermediate Scattering Function

$$F(q, t) = \frac{1}{N} \left\langle \sum_{i=1}^N \sum_{j=1}^N e^{i\vec{q} \cdot (\vec{r}_i(t) - \vec{r}_j(0))} \right\rangle$$



► Damped Sound Wave

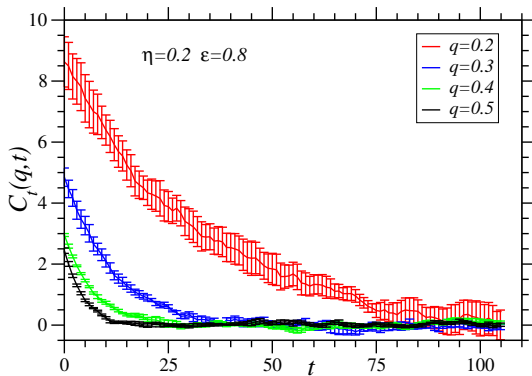
► Simplified Model:

$$F(q, t) = \frac{\xi_0^2 h}{4c^2 \Gamma q^2} e^{-\Gamma q^2 t} \cos(cqt)$$

► work in progress for more statistics

Transverse Current Correlation

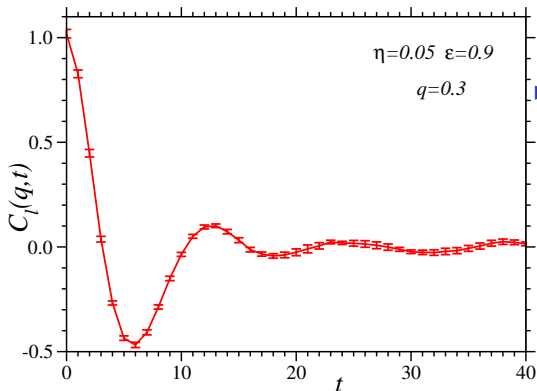
$$C_t(q, t) = \frac{1}{2N} \left\langle \sum_{i=1}^N \sum_{j=1}^N [\hat{q} \times \vec{v}_i(t)] \cdot [\hat{q} \times \vec{v}_j(0)] e^{i\vec{q} \cdot (\vec{r}_i(t) - \vec{r}_j(0))} \right\rangle$$



- ▶ with increasing q decreasing $C_t(q, 0)$
- ▶ with increasing q faster decay

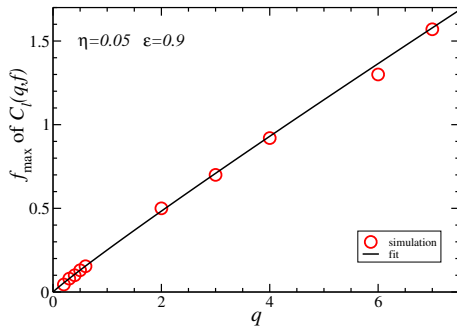
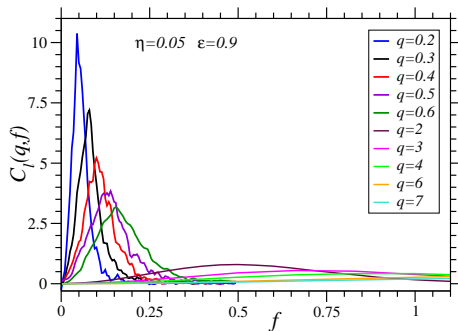
Longitudinal Current Correlation Function

$$C_l(q, t) = \frac{1}{N} \left\langle \sum_{i=1}^N \sum_{j=1}^N \hat{q} \cdot \vec{v}_i(t) \hat{q} \cdot \vec{v}_j(0) e^{i\vec{q} \cdot (\vec{r}_i(t) - \vec{r}_j(0))} \right\rangle$$



► damped soundwave

Spectrum of Longitudinal Current Fluctuations



- ▶ dispersion relation linear for small q
- ▶ fit: $f_{\max} \propto q^{0.94}$

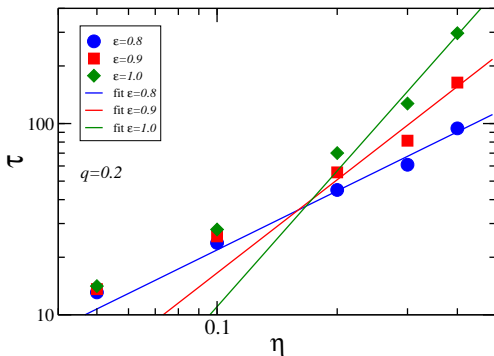
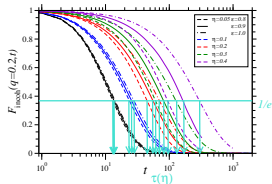
Summary

- ▶ $F_{\text{incoh}}(q, t)$:
 - ▶ Gaussian
 - ▶ $\tau(\eta)$ divergence
 - ▶ time-superposition
- ▶ Damped Soundwaves ($F(q, t), C_l(q, t)$)

Acknowledgments:

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Relaxation Time



- ▶ τ incr. with incr. η
- ▶ compare: $\eta_{\text{glass}} = 0.58$
- ▶ for $\epsilon = 0.8$ $\tau \propto \eta^{1.6}$