Correlation Functions of a Homogeneously Driven Granular Fluid in Steady State

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March 18, 2009

Model & Simulation

- Hard Spheres, 3 dim.
- Dissipation

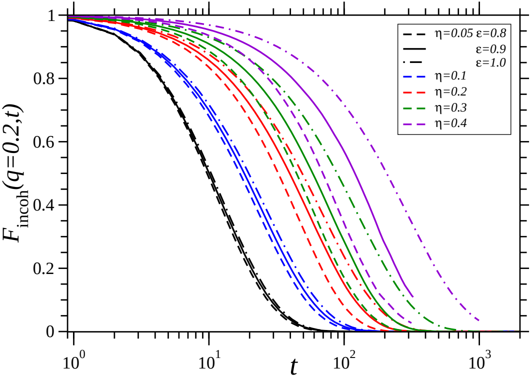
  $$\vec{n} \cdot \left(\vec{v_1}' - \vec{v_2}'\right) = -\epsilon \vec{n} \cdot (\vec{v_1} - \vec{v_2})$$

  $\epsilon =$ coefficient of normal restitution
- Nonequilibrium Steady State
- Volume Driving
  - $\frac{\mathrm{d}}{\mathrm{d}t}\vec{v_i} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\vec{v_i}\right)_{\mathrm{coll}} + \vec{\xi_i}(t)$ [van Noije et al. 1999]
  - $\xi_i(t)$ Gaussian white noise with

    $\langle \vec{\xi} = 0 \rangle$ and $\langle \xi_{i\alpha}(t) \xi_{j\beta}(t') \rangle = \xi_0^2 \delta_{ij} \delta_{\alpha\beta} \delta(t - t')$
  - to conserve total momentum globally fixed pairs with opposite kicks
- Event Driven Simulations
  - $N = 200000$
  - Volume Fractions $\eta = 0.05, 0.1, 0.2, 0.3, 0.4$
  - $\epsilon = 1.0$ (elastic), 0.9, 0.8

Incoherent Intermediate Scattering Function

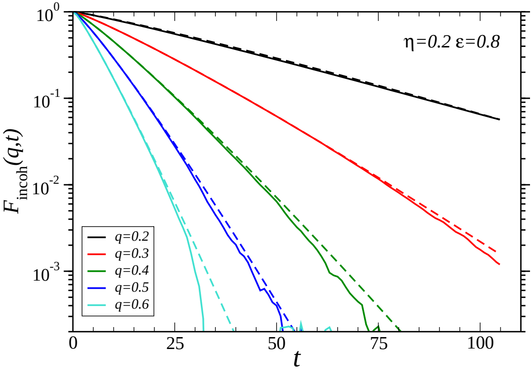
$$F_{\mathsf{incoh}}(q,t) = \langle \frac{1}{N} \sum_{i=1}^{N} \mathsf{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_i(0))} \rangle$$



- dependence on ϵ & η
- not dense enough for glassy behavior
- Gaussian approximation?
- relaxation time τ

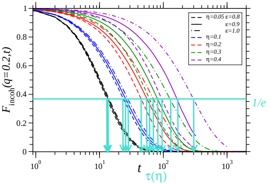
Gaussian Approximation of $F_{\text{incoh}}(q,t)$

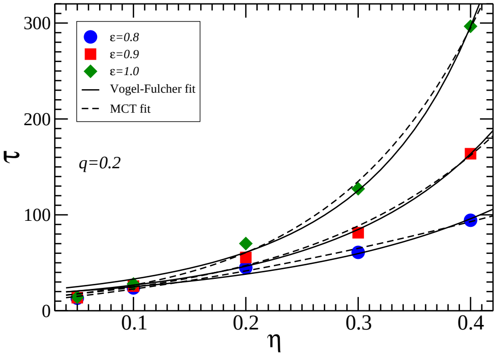
$$F_{\text{incoh}}(q,t) = \mathrm{e}^{-\frac{1}{6}\, q^2 \langle \frac{1}{N} \sum_{i=1}^{N} (\vec{r}_i(t) - \vec{r}_i(0))^2 \rangle}$$



- good approximation
- similarly for other η, ϵ, q

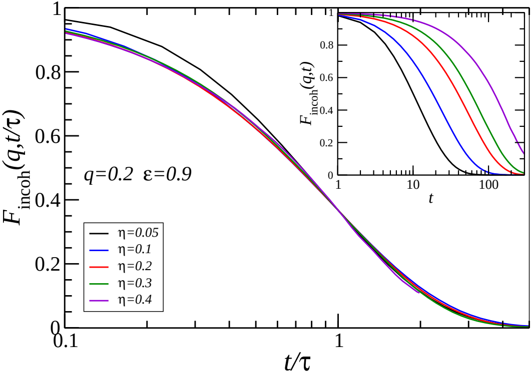
Relaxation Time





- τ rapidly increasing with increasing η
- faster increase for larger ϵ
- compare: $\eta_{\text{glass}} = 0.58$

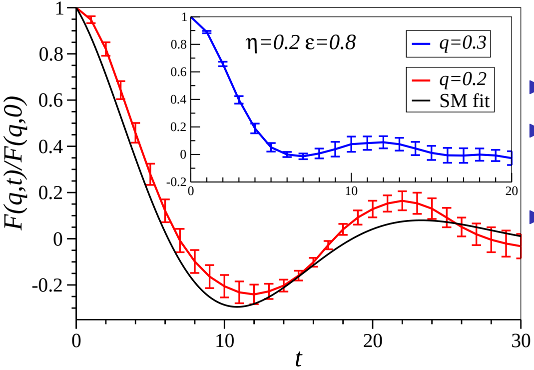
Incoherent Intermediate Scattering Function



- time superposition for all η but smallest
- curves are of same shape
- similarly for other q, ϵ

Intermediate Scattering Function

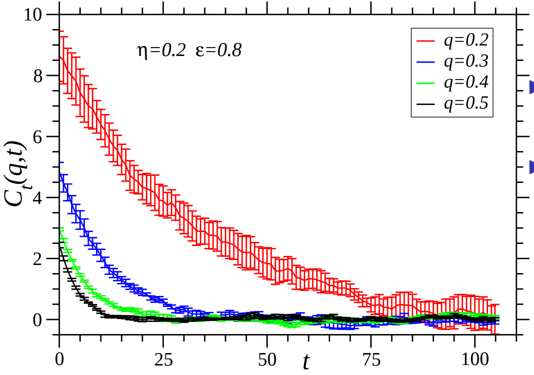
$$F(q,t) = \frac{1}{N}\langle \sum_{i=1}^{N} \sum_{j=1}^{N} \mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))} \rangle$$



- Damped Sound Wave
- Simplified Model: $F(q,t) = \frac{\xi_0^2 h}{4c^2\Gamma q^2}\mathrm{e}^{-\Gamma q^2 t}\cos(cqt)$
- work in progress for more statistics

Transverse Current Correlation

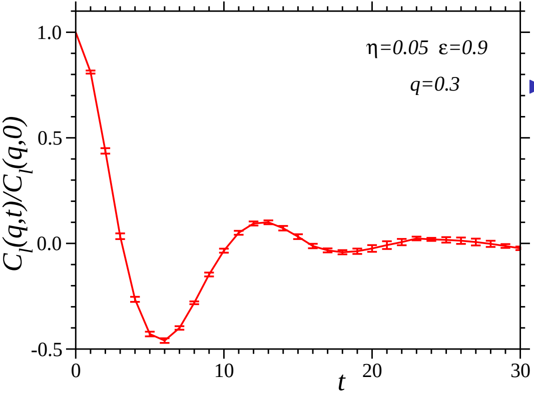
$$C_t(q,t) = \frac{1}{2N}\langle \sum_{i=1}^{N}\sum_{j=1}^{N} [\hat{q}\times\vec{v}_i(t)]\cdot[\hat{q}\times\vec{v}_j(0)]\,\mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))}\rangle$$



- with increasing q decreasing $C_t(q,0)$
- with increasing q faster decay

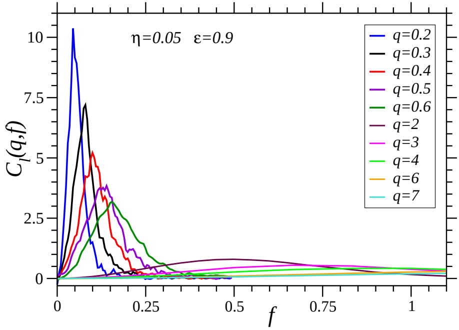
Longitudinal Current Correlation Function

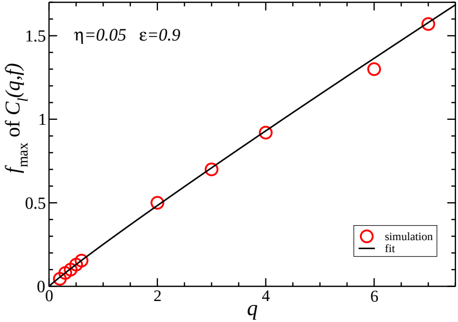
$$C_l(q,t) = \frac{1}{N}\langle \sum_{i=1}^{N}\sum_{j=1}^{N} \hat{q}\cdot\vec{v}_i(t)\,\hat{q}\cdot\vec{v}_j(0)\mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))}\rangle$$



- damped soundwave

Spectrum of Longitudinal Current Fluctuations





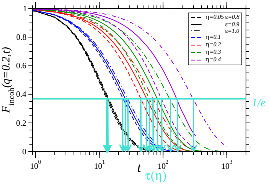
- dispersion relation linear for small q
- fit: $f_{\mathsf{max}} \propto q^{0.94}$

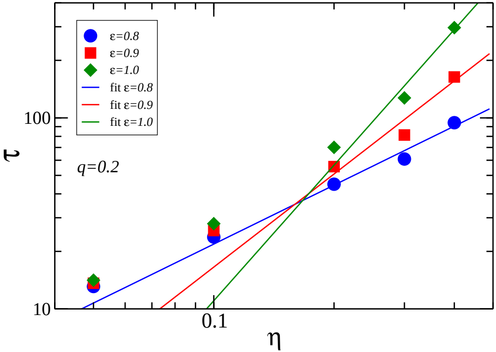
Summary

- $F_{\text{incoh}}(q, t)$:
  - Gaussian
  - $\tau(\eta)$ divergence
  - time-superposition
- Damped Soundwaves $(F(q, t), C_l(q, t))$

Acknowledgments:
Support from Institute of Theoretical Physics, University Göttingen

Relaxation Time

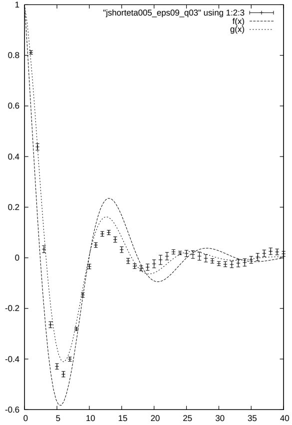




- τ incr. with incr. η
- compare: $\eta_{\text{glass}} = 0.58$
- for $\epsilon = 0.8$ $\tau \propto \eta^{1.6}$

Longitudinal Current Correlation Function

$$C_l(q,t) = \frac{1}{N}\langle \sum_{i=1}^{N}\sum_{j=1}^{N} \hat{q}\cdot\vec{v}_i(t)\,\hat{q}\cdot\vec{v}_j(0)\mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))}\rangle$$



- damped soundwave