Sound Waves in a Homogeneously Driven Granular Fluid in Steady State

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Hard Spheres, 3 dim.

Dissipation
\[ \vec{n} \cdot (\vec{v}_1' - \vec{v}_2') = -\epsilon \vec{n} \cdot (\vec{v}_1 - \vec{v}_2) \]
\[ \epsilon = \text{coefficient of normal restitution} \]

Nonequilibrium Steady State

Volume Driving
\[ \frac{d}{dt} \vec{v}_i = \left( \frac{d}{dt} \vec{v}_i \right)_{\text{coll}} + \vec{\xi}_i(t) \]
\[ \vec{\xi}_i(t) \text{ Gaussian white noise with } \langle \vec{\xi} = 0 \rangle \text{ and } \langle \xi_{i\alpha}(t)\xi_{j\beta}(t') \rangle = \xi_0^2 \delta_{ij} \delta_{\alpha\beta} \delta(t - t') \]
\[ \text{to conserve total momentum globally fixed pairs with opposite kicks} \]

Event Driven Simulations
\[ N = 10000 \]
\[ \text{Volume Fractions } \eta = 0.05, 0.1, 0.2 \]
\[ \epsilon = 0.9, 0.8 \]
\[ \text{each 100 independent simulation runs} \]
Dynamics of Driven Granular Fluid in Steady State

**Intermediate Scattering Function**

\[
F(q, t) = \langle \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} e^{i \vec{q} \cdot (\vec{r}_i(t) - \vec{r}_j(0))} \rangle
\]

**Longitudinal Current Correlation Function**

\[
C_1(q, t) = \frac{1}{N} \langle \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{q} \cdot \vec{v}_i(t) \hat{q} \cdot \vec{v}_j(0) e^{i \vec{q} \cdot (\vec{r}_i(t) - \vec{r}_j(0))} \rangle
\]

Graphs showing the decay of \(F(q, t)/F(q, 0)\) and \(C_1(q, t)/C_1(q, 0)\) with parameters \(\eta=0.05\) and \(\varepsilon=0.8\)

\(\Rightarrow\) damped soundwave
Simulation:
- $C_1(q, t)$
- $C_1(q, f')$
- $f_{\text{max}}(q), \text{HW}(q)$
- $F(q, f)$

Theory:
- simplified model
- model including thermal fluctuations
Longitudinal Current Correlation Function

Damped Soundwave:
- with increasing $\eta$:
  - more damped
  - increasing $\omega_0$
Damped Soundwave:

- with increasing $\eta$:
  - more damped
  - increasing $\omega_0$

- with increasing $q$:
  - more damped
  - increasing $\omega_0$
Longitudinal Current Correlation Function

Damped Soundwave:

- with increasing $\eta$:
  - more damped
  - increasing $\omega_0$

- with increasing $q$:
  - more damped
  - increasing $\omega_0$

- with decreasing $\epsilon$:
  - more damped
  - increasing $\omega_0$
Spectrum of Longitudinal Current Fluctuations

$C_1(q, f)$:
- with increasing $\eta, q$:
  - peak shifts to right
  - width increasing
- little dependence on $\epsilon$:
Dispersion Relation via $C_1(q, f)$

- increasing $f_{\text{max}}$:
  - with increasing $q$
  - with increasing $\eta$
Dispersion Relation via $C_1(q, f)$

- increasing $f_{\text{max}}$: 
  - with increasing $q$ 
  - with increasing $\eta$
- for small $q$ linear
- slope increases with increasing $\eta$
Dispersion Relation via $C_1(q, f)$

- increasing $f_{\text{max}}$
  - with increasing $q$
  - with increasing $\eta$
- for small $q$ linear
- slope increases with increasing $\eta$
- almost independent of $\epsilon$
Width of $C_1(q, f)$

- increasing width:
  - with increasing $q$
  - with increasing $\eta$
- almost independent of $\varepsilon$
Dynamic Structure Factor $F(q, f)$

Damped Soundwave:

- **strongly damped:**
  - for small $\eta$
  - for large $q$
  - for smaller $\epsilon$

- **additional shoulder for:**
  - for large $\eta$
  - for small $q$
  - for larger $\epsilon$
\[ \partial_t \delta n = -ik n_0 u \]
\[ \partial_t u = -\frac{ik p_0}{\rho_0 n_0} \delta n - \nu_1 k^2 u + \xi \]

[van Noije et al., PRE 59, 4326 (1999)]
\[ \partial_t \delta n = -i k n_0 u \]
\[ \partial_t u = -i k \left( \frac{p_0}{n_0} \delta n + \frac{p_0}{T_0} \delta T \right) - \nu_1 k^2 u + \xi \]
\[ \partial_t T = -D_T k^2 \delta T - \frac{2p}{dn_0} k u - \gamma_0 \omega_E \frac{T_0}{n_0} \delta n - 3\gamma_0 \omega_E \delta T + \theta \]

[van Noije et al., PRE 59, 4326 (1999)]
Conclusions

- damped sound waves
- $C_1(q, f)$:
  - $f_{\text{max}}$ increasing with increasing $\eta, q$
  - $f_{\text{max}}$ linear for small $q$
  - width increasing with increasing $\eta, q$
  - only small dependence on $\epsilon$
- hydrodynamic model:
  - good fit for $F(q, f)$
  - thermodynamic fluctuations present

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