Sound Waves in a Homogeneously Driven Granular Fluid in Steady State

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Model & Simulation

- Hard Spheres, 3 dim.
- Dissipation

$$\vec{n} \cdot \left(\vec{v_1}' - \vec{v_2}'\right) = -\epsilon \vec{n} \cdot (\vec{v_1} - \vec{v_2})$$

$\epsilon =$ coefficient of normal restitution

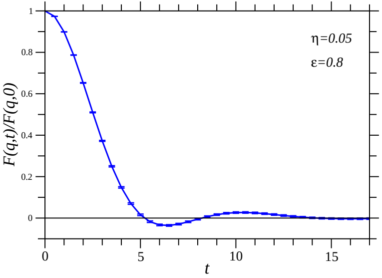
- Nonequilibrium Steady State
- Volume Driving
  - $\frac{\mathrm{d}}{\mathrm{d}t}\vec{v_i} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\vec{v_i}\right)_{\mathrm{coll}} + \vec{\xi_i}(t)$ [van Noije et al. 1999]
  - $\xi_i(t)$ Gaussian white noise with

    $\langle\vec{\xi} = 0\rangle$ and $\langle \xi_{i\alpha}(t)\xi_{j\beta}(t')\rangle = \xi_0^2 \delta_{ij}\delta_{\alpha\beta}\delta(t-t')$
  - to conserve total momentum globally fixed pairs with opposite kicks
- Event Driven Simulations
  - $N = 10000$
  - Volume Fractions $\eta = 0.05, 0.1, 0.2$
  - $\epsilon = 0.9,\ 0.8$
  - each 100 independent simulation runs

Dynamics of Driven Granular Fluid in Steady State

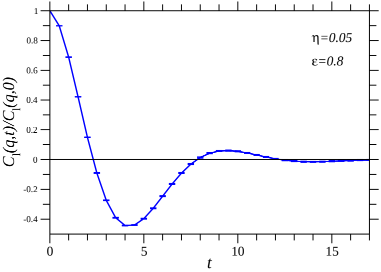
Intermediate Scattering Function

$$F(q,t) = \langle \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))} \rangle$$



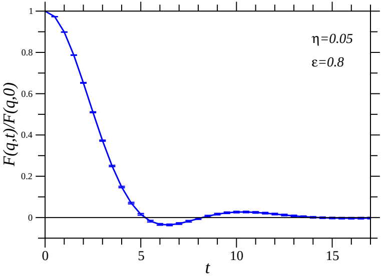
Longitudinal Current Correlation Function

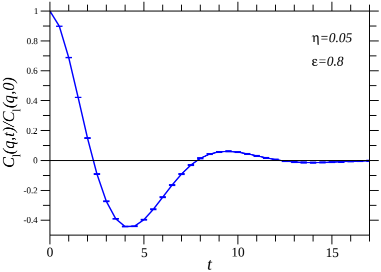
$$C_\mathrm{l}(q,t) = \frac{1}{N} \langle \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{q}\cdot\vec{v}_i(t)\,\hat{q}\cdot\vec{v}_j(0)\mathrm{e}^{i\vec{q}\cdot(\vec{r}_i(t)-\vec{r}_j(0))} \rangle$$



$\Longrightarrow$ damped soundwave

Outline





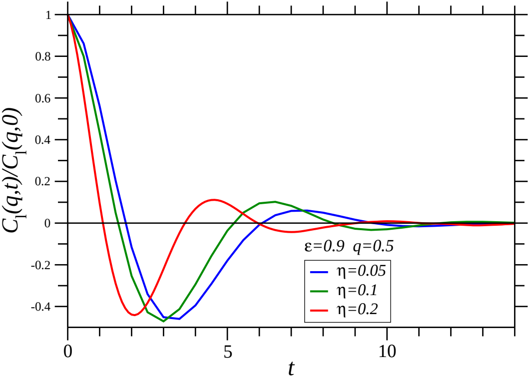
Simulation:

- $C_\mathrm{l}(q,t)$
- $C_\mathrm{l}(q,f)$
- $f_\mathrm{max}(q)$,HW(q)
- $F(q,f)$

Theory:

- simplified model
- model including thermal fluctuations

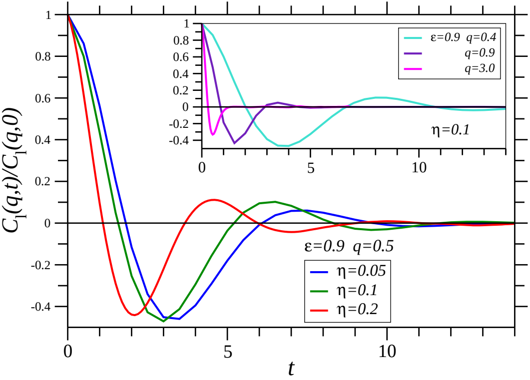
Longitudinal Current Correlation Function



Damped Soundwave:

- with increasing η:
  - more damped
  - increasing ω_0

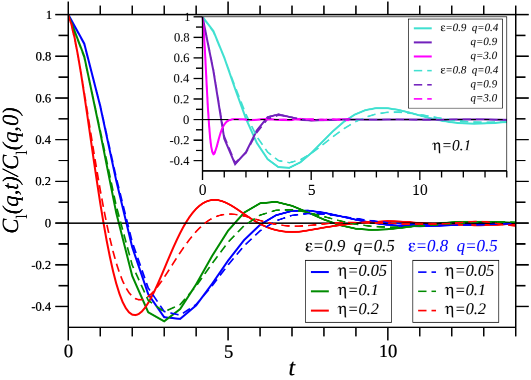
Longitudinal Current Correlation Function



Damped Soundwave:

- with increasing η:
  - more damped
  - increasing ω_0
- with increasing q:
  - more damped
  - increasing ω_0

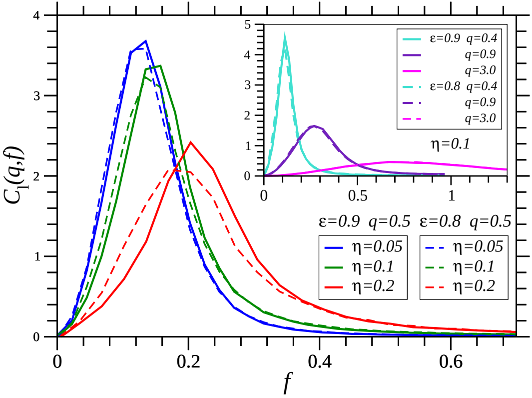
Longitudinal Current Correlation Function



Damped Soundwave:

- with increasing η:
  - more damped
  - increasing ω_0
- with increasing q:
  - more damped
  - increasing ω_0
- with decreasing ϵ:
  - more damped
  - increasing ω_0

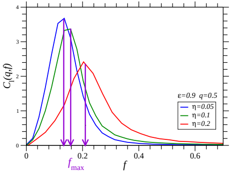
Spectrum of Longitudinal Current Fluctuations

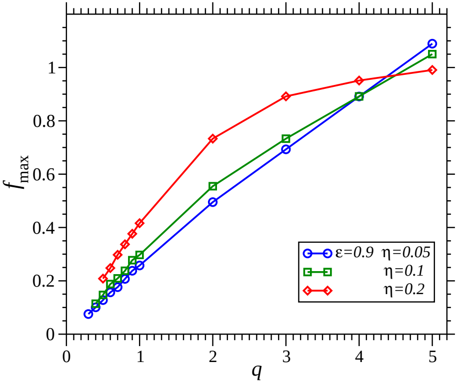


$C_{\rm l}(q, f)$:

- with increasing η,q:
  - peak shifts to right
  - width increasing
- little dependence on ϵ:

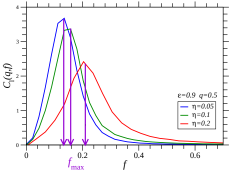
Dispersion Relation via $C_{\mathrm{l}}(q, f)$

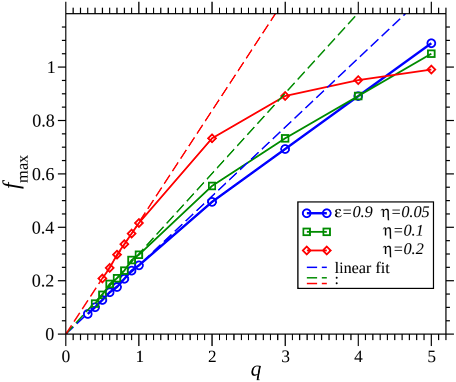




- increasing f_{max}:
  - with increasing q
  - with increasing η

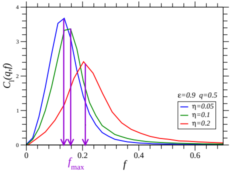
Dispersion Relation via $C_l(q, f)$

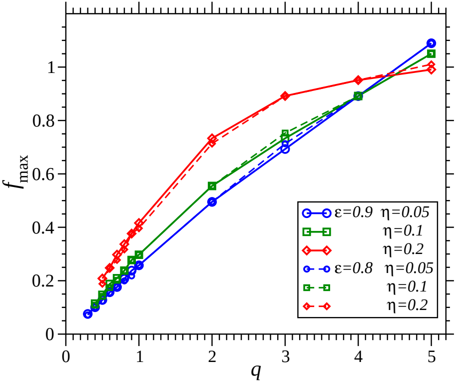




- increasing $f_{\max}$:
  - with increasing q
  - with increasing η
- for small q linear
- slope increases with increasing η

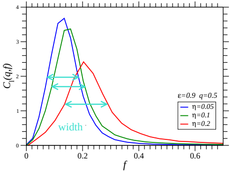
Dispersion Relation via $C_\mathrm{l}(q, f)$

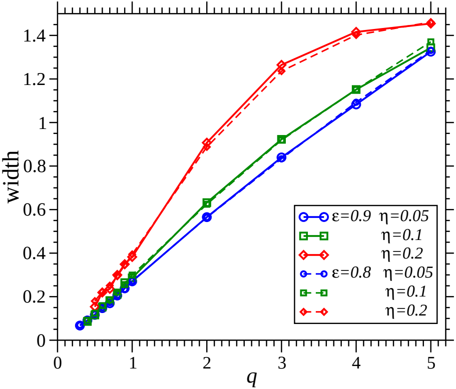




- increasing $f_{\max}$:
  - with increasing q
  - with increasing η
- for small q linear
- slope increases with increasing η
- almost independent of ϵ

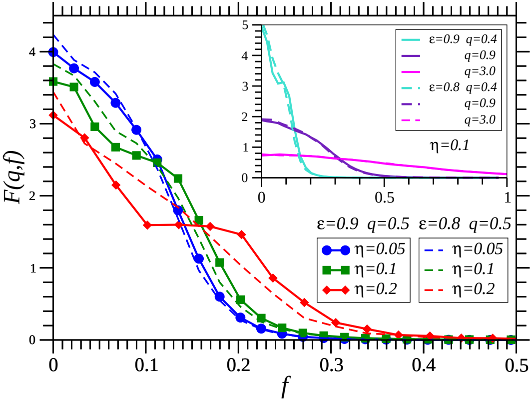
Width of $C_{\mathrm{l}}(q, f)$





- increasing width:
  - with increasing q
  - with increasing η
- almost independent of ϵ

Dynamic Structure Factor $F(q, f)$



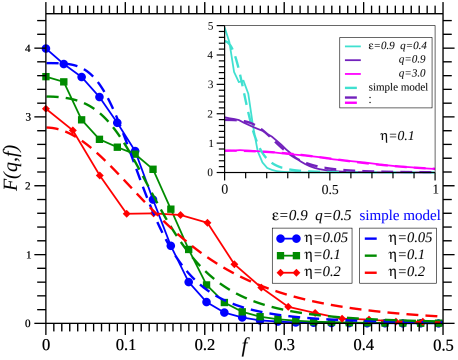
Damped Soundwave:

- strongly damped :
  - for small η
  - for large q
  - for smaller ϵ
- additional shoulder for:
  - for large η
  - for small q
  - for larger ϵ

Simplified Model Without Coupling to Temperature

$$
\begin{aligned}
\partial_t \delta n &= -ik\, n_0\, u \\
\partial_t u &= -\frac{ik}{\rho_0}\frac{p_0}{n_0}\delta n - \nu_l\, k^2\, u + \xi
\end{aligned}
$$

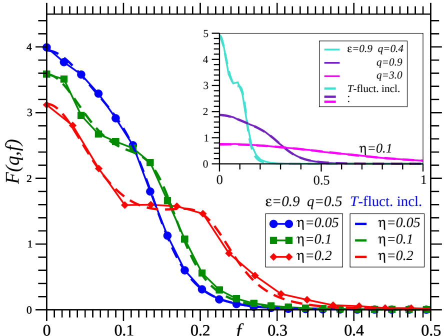
[van Noije et al., PRE **59**, 4326 (1999)]



Extended Model With Coupling to Temperature

$$\begin{aligned}
\partial_t \delta n &= -ik\, n_0\, u \\
\partial_t u &= -\frac{ik}{\rho_0}\left(\frac{p_0}{n_0}\delta n + \frac{p_0}{T_0}\delta T\right) - \nu_1\, k^2\, u + \xi \\
\partial_t T &= -D_T\, k^2\, \delta T - \frac{2p}{dn_0}\, k\, u - \gamma_0 \omega_{\mathrm{E}} \chi \frac{T_0}{n_0}\, \delta n - 3\gamma_0 \omega_{\mathrm{E}}\, \delta T + \theta
\end{aligned}$$

[van Noije et al., PRE **59**, 4326 (1999)]



Conclusions

- damped sound waves
- $C_l(q, f)$:
  - $f_{\max}$ increasing with increasing η, q
  - $f_{\max}$ linear for small q
  - width increasing with increasing η, q
  - only msall dependence on ϵ
- hydrodynamic model:
  - good fit for $F(q, f)$
  - thermodynamic fluctuations present

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