R. J. Gaylord & P. R. Wellin,
"Computer Simulations with Mathematica",
Springer-Verlag, New York 1995.

Cellular Automata Preliminaries

WHAT IS A CELLULAR AUTOMATON?

A *cellular automaton* (or CA) consists of a discrete system of lattice sites having various initial values. These sites evolve in discrete time steps as each site assumes a new value based on the values of some local neighborhood of sites and a finite number of previous time steps.

Before starting to work with cellular automata models, we need to provide some background. We will define the lattices that we'll be using and show the neighborhoods of lattice sites for various boundary conditions.

CA LATTICES

In one dimension, a cellular automaton is a simple linear list of the following form (where `expr` evaluates to numbers and/or symbols):

```
Table[expr, {i, 1, s}]
```

In two dimensions, various lattices can be used (*e.g.*, rectangular, triangular, hexagonal). We will be using rectangular $n \times m$ lattices that can be written using the `Table` function.

```
Table[expr, {i, 1, n}, {j, 1, m}]
```

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CHAPTER 8

The Game of Life

INTRODUCTION

The Game of Life, created by the British mathematician John Conway, is the most famous cellular automaton (CA). It has been said that more computer time has been spent on running the Game of Life than on any other computation. The Game of Life was the first program run on the Connection Machine, the world's first parallel computer. Most importantly, it is the forerunner of so-called *artificial life* (or *a-life*) systems which are of great interest today, not only for their biological implications, but for the development of so-called "intelligent agents" for computers.

8.1 THE GAME OF LIFE

The Game of Life is played on a two-dimensional square lattice with periodic boundary conditions. Lattice sites have a value of 0 or 1 (*i.e.*, the lattice is a Boolean matrix). A site with value 1 is said to be *alive* and a site with value 0 is said to be *dead*. The system evolves by updating all of the sites in the lattice simultaneously, based on their Moore neighborhoods, until two successive lattice configurations are identical, or until a specified number of updates (time steps) have occurred.

THE LIFE RULES

The Game of Life CA rules are based on the value of a site and the sum of the values of its neighbors. The rules, known as *life and death* rules, are:

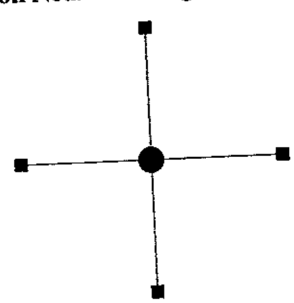
- A living site with two living nearest neighbor sites remains alive.
- Any site with three living nearest neighbor sites stays alive or is born.
- All other sites either remain dead or die.

CA Neighborhoods

The *neighborhood* of a lattice site consists of the site and its nearest neighbor sites (called *neighbors*). Two kinds of neighborhoods are commonly defined for the rectangular lattice.

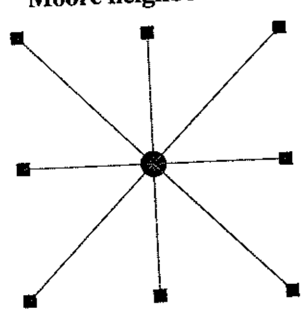
- A **von Neumann** neighborhood consists of the site and the four nearest neighbors, north (above), east (right), south (below), and west (left) of the site. In the graphics below, the site is represented by a filled disk and its neighbors by gray squares.

von Neumann neighborhood



- A **Moore** neighborhood consists of the site and the eight nearest neighbor sites, north, northeast, east, southeast, south, southwest, west, and northwest of the site.

Moore neighborhood



The nearest neighbors of sites along the sides (borders) of a lattice are determined differently for various boundary conditions. To illustrate these different criteria, the corresponding Moore neighborhoods are shown below for each site in the following simple lattice:

```
1    2    3
4    5    6
7    8    9
10   11   12
```

Periodic Boundaries

- The nearest neighbor left of a site on the left border is the site in the same row on the right border.
- The nearest neighbor right of a site on the right border is the site in the same row on the left border.
- The nearest neighbor above a site on the top border is the site in the same column on the bottom border.
- The nearest neighbor below a site on the bottom border is the site in the same column on the top border.

```
12 10 11    10 11 12    11 12 10
3  1  2     1  2  3     2  3  1
6  4  5     4  5  6     5  6  4

3  1  2     1  2  3     2  3  1
6  4  5     4  5  6     5  6  4
9  7  8     7  8  9     8  9  7

6  4  5     4  5  6     5  6  4
9  7  8     7  8  9     8  9  7
12 10 11    10 11 12    11 12 10

9  7  8     7  8  9     8  9  7
12 10 11    10 11 12    11 12 10
3  1  2     1  2  3     2  3  1
```

As the table above shows, the neighbors of site 4 are 3, 1, 2, 6, 5, 9, 7, and 8.

Absorbing Boundaries

- The nearest neighbor left of a site on the left border is zero.
- The nearest neighbor right of a site on the right border is zero.
- The nearest neighbor above a site on the top border is zero.
- The nearest neighbor below a site on the bottom border is zero.