## In-Class Work: Traffic Flow

## 1. Uniform Random Numbers

1a. Copy the program
~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/float_rand0-1.cc
into your working directory. Compile the program and let it run. Scan the header and main of the program to learn how to get random numbers. Do not try to understand the function randomd itself. Random function generators are an art for itself, and we just use this random number generator because it is a very well working one. Whenever you want to write a program in which you use random numbers you need to do the following steps:

1. Include in the header of your program the two lines:
```
double randomd(long *);
    long idummy = -7;
```

2. Include at the end of your program the definition of the function randomd, i.e. lines 25-57 of float_rand0-1.cc.
3. whenever you want another random number use randomd (\&idummy)

1b. Have a look at which random numbers you get:
float_rand0-1.out
and to see the numbers graphically
float_rand0-1.out | xmgrace -pipe
or if you want a quick way to look at the data points not connected with lines float_rand0-1.out | xgraph -m -nl

## 2. Initialize Car Positions

2a. Use the program float_rand0-1.cc to write a program that prints (instead of random numbers) 100 numbers where each number is " 1 " with probability PCAR $=0.3$ and otherwise " -1 ". Run the program and check it for example with xgraph as in 1 b .

2b. Use your program from 2a. and use it to initialize a road array of ROADLENGTH=20. Put on each site a car with probability $\operatorname{PCAR}=0.3$ by assigning a value 1 and otherwise keep the road site empty by assigning the value -1 . Check the program by printing the road.

## 3. Initialize Road

3a. For the initialization of the road we also will have to initialize the velocity of each car. As preparation for this task modify your program of 2a such that it prints 500 random numbers $v \in\{0,1, \ldots$, VMAX $\}$ (use VMAX=5) by using the random number generator. Hint: You may use that int converts double to integer, for example $\operatorname{int}(4.67)=4$ and $\operatorname{int}(3.05)=3$.

3b. Now you are ready to initialize the road. Modify your program of $2 b$ to put on each site of the road array a car with probability PCAR and give each car on the road a random velocity $v \in\{0,1, \ldots$, VMAX $\}$ (use VMAX=5).

## 4. Distance (if time)

Please get me before you start working on the following so that I can explain the next steps.
4a. Next we work on finding the distance between a car and the car in front of it. To get this distance we will avoid in the following to have to check each site in front of a car if it is empty or not. Instead we define an additional array: carpos which stores for each car on the road its position on the road. So if the first car (index 0 ) on the road is on site 3 then carpos [0]=3, if the second car is on site 5 then carpos [1]=5, etc.. Add to your program of 3b. the array carpos, initialize carpos in the same loop when you initialize the road array, and check your program by printing out both the complete road and the carpos array.

4b. Now add to your program (after the initialization of road and carpos) a loop over cars on the street determine (and print out) for each car the distance to the car in front of it. Take into accout the periodic boundary conditions both to determine the car in front and to determine the distance.
Comment: This task is more difficult than it might seem at first glance. Please get me after having thought about this task.

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## 4. Distance

Copy the following solution program which determines the distance $d$ for each car on the road ~kvollmay/classes.dir/capstone_s2012.dir/traffic5_sample.cc
or if you had at the end of last class a working program for this task, then you may use your program from last class.

## 5. Update Velocities

5a. For the update of the velocities we will need a function which determines the smallest number of three integers. Please note that traffic5_sample.cc already has included a function which reads in three integers, determines via a function which of these three integers the smallest number is, and prints out the result. Test this function by printing the minimum of each of the three sets: $(3,2,10),(1,13,10),(16,14,10)$ (see comments in program).
5b. Use the program traffic5_sample.cc and add to it that it determines all new velocities (and updates them in the road array), and prints out the road with the new velocities. Compare your result for the case
(idummy $=-7$, ROADLENGTH $=20, \mathrm{VMAX}=5, \mathrm{PCAR}=0.3$ ) with
~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic5b.data

## 6. Update Positions

Now you are ready to add to your program the update of the positions. To do so use a new loop over all cars and update both the carpos array and the road array. For the update of road make sure to first copy the new velocity into a variable (e.g. vnew), then empty the old site and then put the car in road on its new site xnew $=$ xold + vnew with the new velocity (the order of these commands matters). After the update of all positions print the complete road //(already in traffic5_sample.cc) and check if it is what you expected. Using the same parameters as given in 5 b compare your result with ~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic6.data Test your program further by using $\mathrm{PCAR}=0.5$ and compare again with ~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic6.data

## 7. Finish Program (if time)

Now you are set to finish the program for our traffic flow model. Add to your program from 6. the time loop. Compare your result for the case of 100 timesteps and otherwise the same parameters as in 5b. Compare your output with
~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic7.data

## 8. Space-Time Diagrams (IF TIME)

8a. Now we are ready to have a look at the flow of the cars. To be able to see the main patterns (and to get nice pictures) use a larger ROADLENGTH (for example =200). We use DynamicLattice to make a graph of time over position. Print the road for every time step, as it was done to get traffic7. data. In case you run your program for 100 timesteps, you get 101 lines, so your DynamicLattice command is for example:
traffic8.out | DynamicLattice -nx 200 -ny 101 -matrix
8b. To distinguish stopped cars easily from driving cars, let's indicate every empty site instead of with -1 now with -VMAX. This results in DynamicLattice in empty sites being blue, stopped cars in white and driving cars in pinkish/red. Look at the space-time diagram.

8c. Vary PCAR. Interpret the resulting space-time diagrams. Once you have several spacetime diagrams for different PCAR, get me so that we can discuss as a class your results.

## 9. Nagel-Schreckenberg Model

9a. Get me when you get to this part. We are now ready to finish the programming of the Nagel-Schreckenberg model. Add to your program of 8. the randomization of the velocity, so complete the Nagel-Schreckenberg Model. Use VMAX = 5, PCAR = 0.2, PDEC = 0.25, ROADLENGTH $=20$, MAXTSTEPS $=8$ and idummy $=-7$ and compare your result with ~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic9a.data
Comment: There are variations in how you might use the random numbers. Therefore, please get me in case of discrepancies between your and my results.
9b. Now set ROADLENGTH=200 and MAXSTEPS=100 and have a look at the resulting space-time diagram with DynamicLattice.
9c. Keep all parameters as in 9b but vary
(i) PCAR between 0.05 and 0.35
(ii) PDEC between 0.0 and 0.5
(iii) VMAX between 1 and 10 .

What do you observe in each case? Please get me to discuss your observations and to share your results with the class.

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## 8. Space-Time Diagrams

## 8a. Copy

~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic8.cc
and have a look at the program. (If you had already finished 8. in last class you may use your own program.) Our first analysis of the traffic flow are the space-time diagrams which you saw in last class. To be able to see the main patterns (and to get nice pictures) use a larger ROADLENGTH (for example $=200$ ). We use DynamicLattice to make a graph of time over position. The program traffic8.cc prints the road for every time step. In case you run your program for 100 timesteps, you get 101 lines, so your DynamicLattice command is for example:
traffic8.out | DynamicLattice -nx 200 -ny 101 -matrix
8b. To distinguish stopped cars easily from driving cars, let's indicate every empty site instead of with -1 now with -VMAX. (Note, that the constant EMPTY defines the value for empty sites.) This results in DynamicLattice in empty sites being blue, stopped cars in white and driving cars in pinkish/red. Look at the space-time diagram.
8c. Vary PCAR. Interpret the resulting space-time diagrams. (Convince yourself of the interpretation we discussed in last class.)

## 9. Nagel-Schreckenberg Model

9a. You are now ready to finish the programming of the Nagel-Schreckenberg model. Add to the program traffic8.cc the randomization of the velocity, so complete the NagelSchreckenberg Model. Use VMAX $=5$, PCAR $=0.2$, PDEC $=0.25$, ROADLENGTH $=20$, MAXTSTEPS=8 and idummy $=-7$ and compare your result with
~kvollmay/classes.dir/capstone_s2011.dir/traffic.dir/traffic9a.data
Comment: There are variations in how you might use the random numbers. Therefore, please get me in case of discrepancies between your and my results.
9b. Now set ROADLENGTH=200 and MAXSTEPS=100 and have a look at the resulting space-time diagram with DynamicLattice.
9c. Keep all parameters as in 9b but vary
(i) PCAR between 0.05 and 0.35
(ii) PDEC between 0.0 and 0.5
(iii) VMAX between 1 and 10 .

What do you observe in each case? Interpret your results. Once you have looked at each case (i)-(iii) please get me to discuss your observations and to share your interpretations with the class. I will then also give you an introduction to the next step 10.

## 10. Mean Velocity $v_{\text {av }}(t)$

Use your program from 9. with parameters (idummy=-7, VMAX=5, PCAR=0.2, PDEC=0.25, ROADLENGTH=1000, and MAXTSTEPS=200) and instead of printing out the road print on the screen for each time $t$ one line with two numbers: t and $v_{\mathrm{av}}$ where $v_{\mathrm{av}}$ is the mean velocity:

$$
\begin{equation*}
v_{\mathrm{av}}=\frac{1}{N} \sum_{i=0}^{N-1} v_{i} \tag{1}
\end{equation*}
$$

is the number of cars and $v_{i}$ is the velocity of car $i$. Compare your result with ~kvollmay/classes.dir/capstone_s2012.dir/traffic.dir/traffic10.data Look at $v_{\mathrm{av}}(t)$ with:
traffic10.out | xmgrace -pipe
Get me so that we can discuss your result. Our interpretation is necessary for 11.
11. $\mathbf{v e q}_{\mathbf{e q}}(c)$ (if time)

For simplicity let us go back to the model without randomized velocities, so use PDEC $=0.0$. Set also VMAX $=4$ and ROADLENGTH $=1000$. You saw in 10. that vav equilibrates after some time to some value $v_{\text {eq }}$ around which $v_{\mathrm{av}}$ fluctuates. We therefore define this equilibrium velocity as

$$
\begin{equation*}
v_{\mathrm{eq}}(c)=\frac{1}{\left(t_{\mathrm{tot}}-t_{\mathrm{eq}}\right)} \sum_{t>\mathrm{t}_{\mathrm{eq}}}^{t_{\mathrm{tot}}} v_{\mathrm{av}}(t) \tag{2}
\end{equation*}
$$

We now want to see how $v_{\text {eq }}$ depends on the concentration of cars

$$
\begin{equation*}
c=\text { nocars/double(ROADLENGTH) } \tag{3}
\end{equation*}
$$

11 a. Take out of your program from 10. the printing of $v_{\mathrm{av}}$. Add to the program that you measure $v_{\text {av }}$ only if the time $t$ is larger than EQUILSTEPS $=100$. Average over these measured $\mathrm{v}_{\mathrm{av}}$ which gives you $v_{\text {eq }}$. Print out the concentration $c$ and $v_{\mathrm{eq}}$. With your program of 10. (adjust PDEC \& VMAX) check if your result of $v_{\text {eq }}$ seems plausible.
11 b . Now change in your program to use PCAR no longer as constant but instead add a loop over pcar (0.1-1.0) and print out for each pcar the concentration $c$ and $v_{\text {eq }}$ as you did in 11a. Have a look at your result with xmgrace
Get me, so that we can discuss the result.
11 c. Set $\mathrm{PDEC}=0.0$ and run your program first for $\mathrm{VMAX}=4$ and then for $\mathrm{VMAX}=2$. In each case redirect the output into a file:
traffic11.out > filename Then set PDEC $=0.25$ and run and redirect the output again for $\mathrm{VMAX}=2$ and $\mathrm{VMAX}=4$. Look at the data of the four files with xgraph.
12. Flux (IF TIME) Get me before you work on this. Next add to your program the flux $j=c \star v_{\text {eq }}$ so that it prints out $c, v_{\text {eq }}$, and the flux $j$. Redirect your output into a file and have a look at your result for $j(c)$ with xmgrace Use the same parameters as in 11 c. Let's look together at your result.

