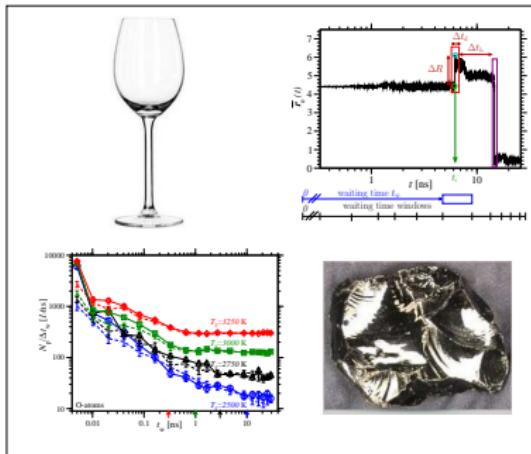


Microscopic Picture of Aging in SiO_2 : A Computer Simulation

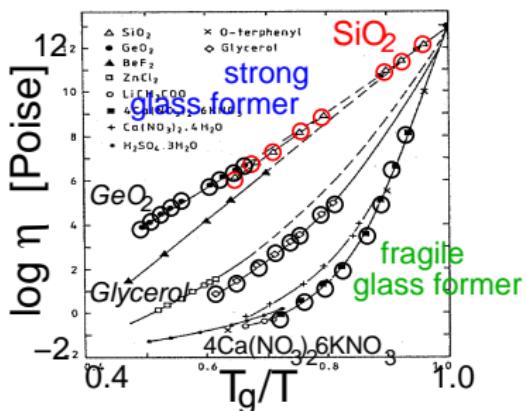
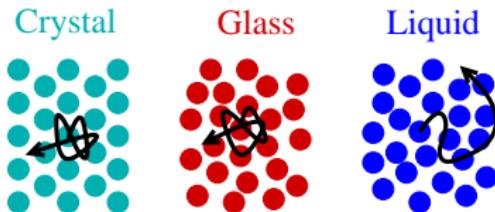
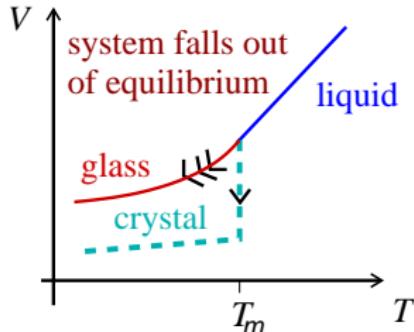
Katharina Vollmayr-Lee, Robin Bjorkquist, Landon M. Chambers
Bucknell University & Göttingen



Mainz, June 25, 2013

Acknowledgments: J. Horbach & A. Zippelius

Introduction: Glass



Dynamics:

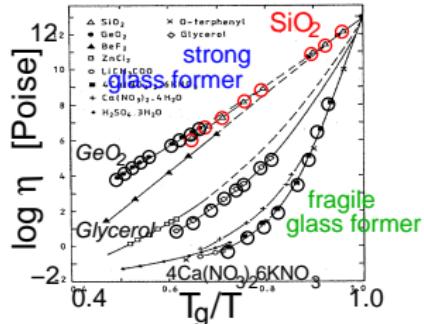
Viscosity η as function of inverse temperature T

- ▶ slowing down of many decades
→ very interesting dynamics
 - ▶ strong and fragile glass formers
- Here: **SiO₂ (strong glass former)**
Below: comparison with fragile glass former

System: SiO₂

Properties:

- rich phase diagram (like H₂O)
- density maximum
- network former
- **strong glass former**

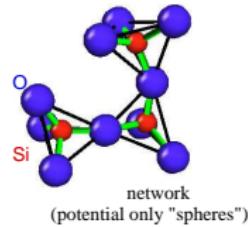


Model: BKS Potential

[B.W.H. van Beest *et al.*, PRL 64, 1955 (1990)]

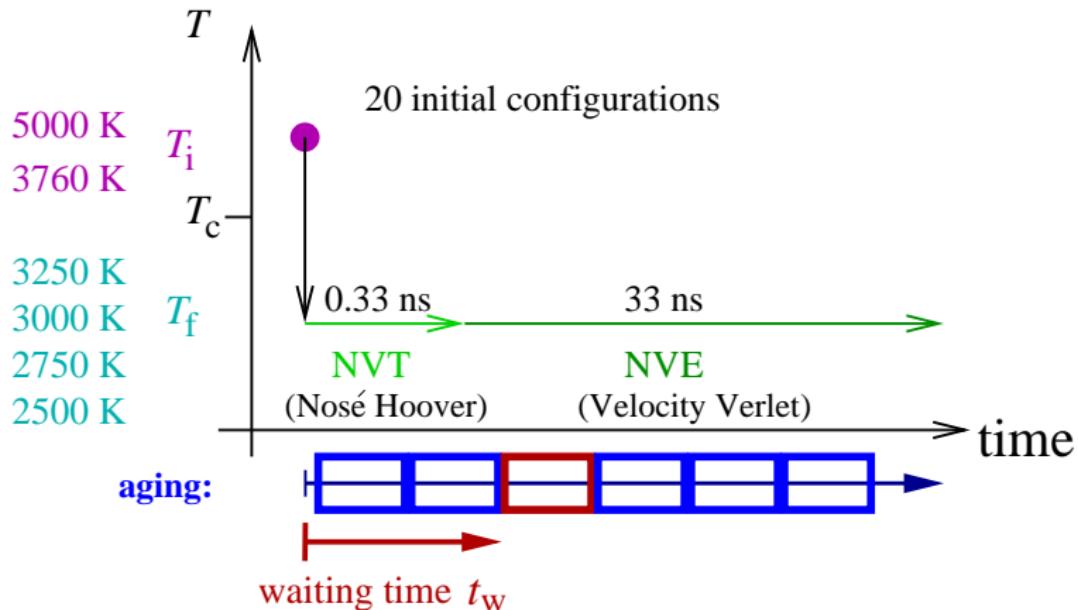
$$\phi(r_{ij}) = \frac{q_i q_j e^2}{r_{ij}} + A_{ij} e^{-B_{ij} r_{ij}} - \frac{C_{ij}}{r_{ij}^6}$$

$$112 \text{ Si} \& 224 \text{ O} \quad \rho = 2.32 \text{ g/cm}^3$$
$$T_c = 3330 \text{ K}$$



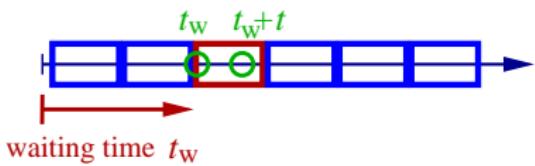
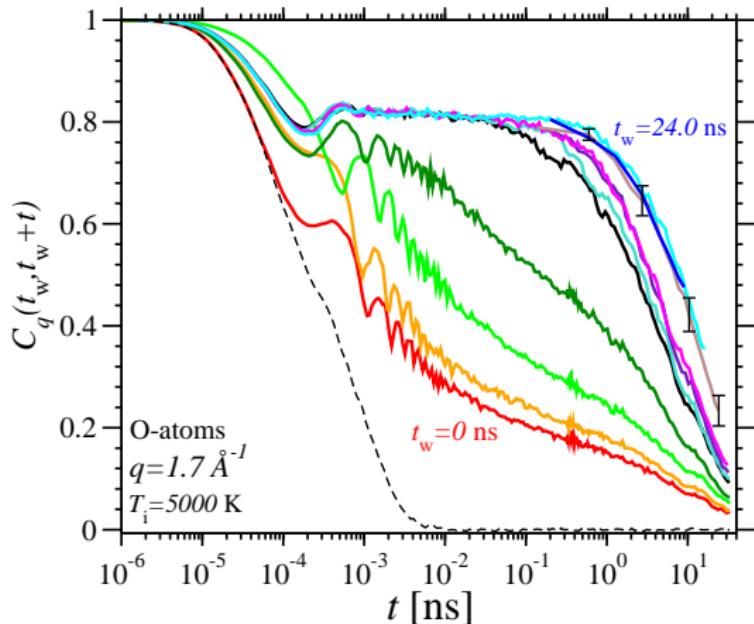
Simulation Runs

Molecular Dynamics Simulations



Generalized Intermediate Incoherent Scattering Function

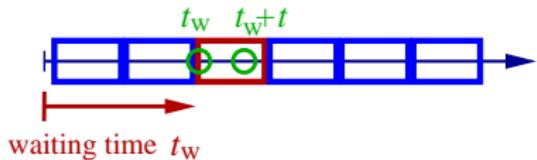
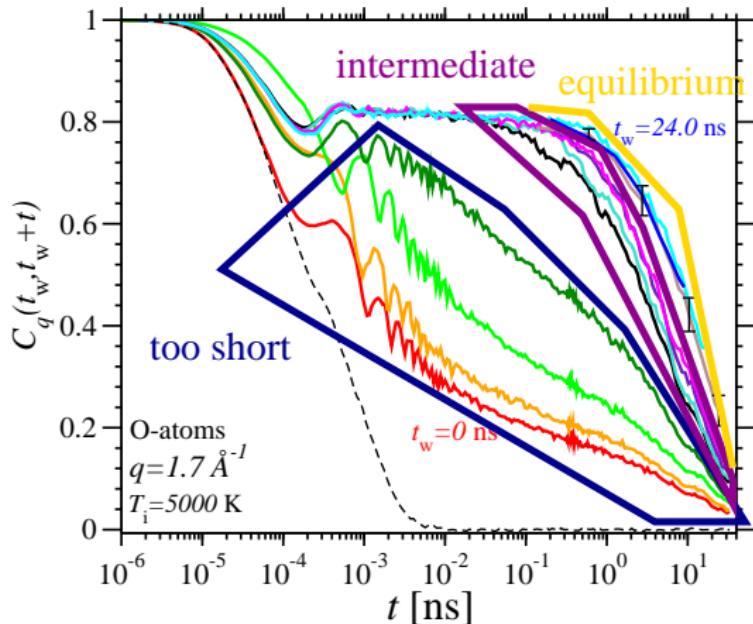
$$C_q(t_w, t_w + t) = \left\langle \frac{1}{N_\alpha} \sum_{j=1}^{N_\alpha} e^{i\vec{q} \cdot (\vec{r}_j(t_w+t) - \vec{r}_j(t_w))} \right\rangle$$



- ▶ $C_q(t_w, t_w + t)$ depends on waiting time t_w (colors)

Generalized Intermediate Incoherent Scattering Function

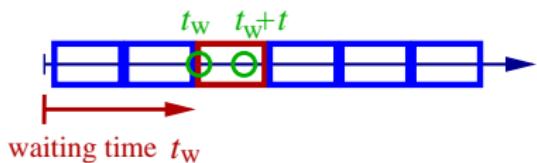
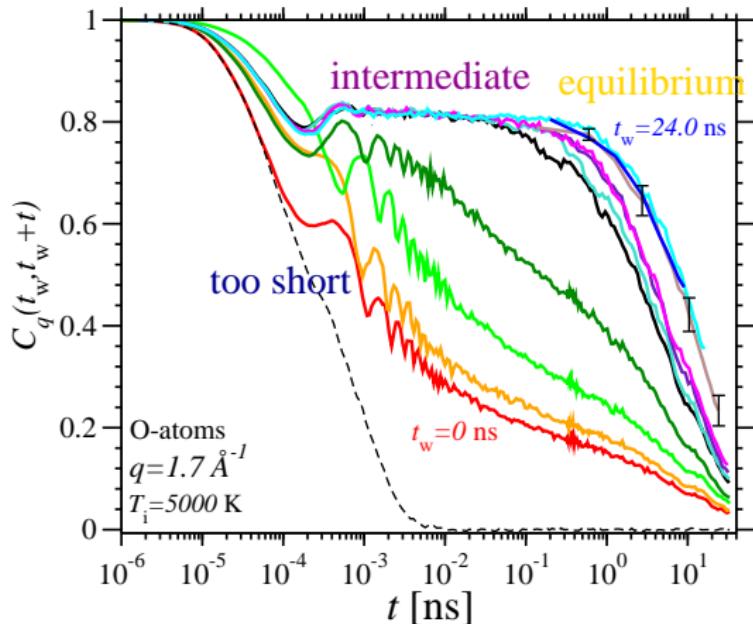
$$C_q(t_w, t_w + t) = \left\langle \frac{1}{N_\alpha} \sum_{j=1}^{N_\alpha} e^{i\vec{q} \cdot (\vec{r}_j(t_w+t) - \vec{r}_j(t_w))} \right\rangle$$



- ▶ three time windows:
 - ▶ too short
 - ▶ intermediate (scaling)
 - ▶ equilibrium (t_{eq}^C)

Generalized Intermediate Incoherent Scattering Function

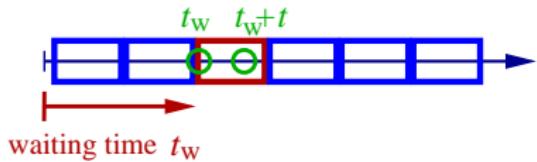
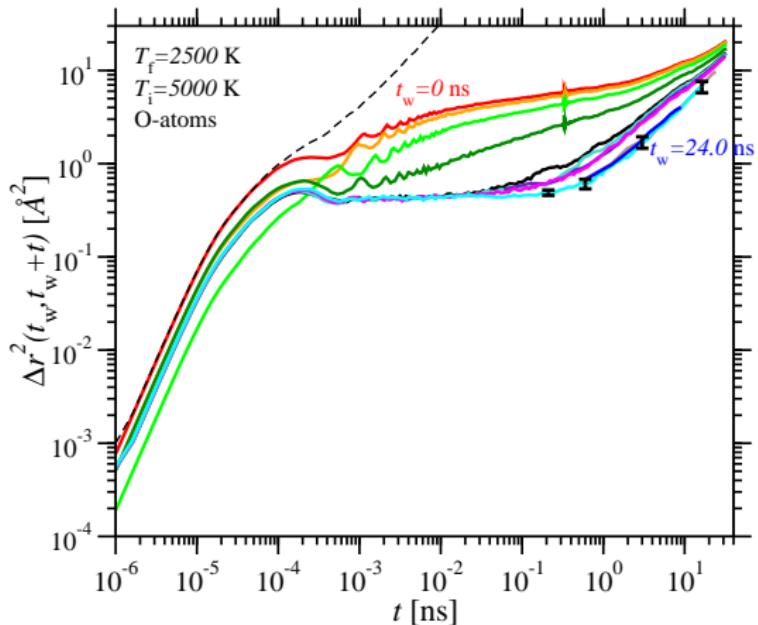
$$C_q(t_w, t_w + t) = \left\langle \frac{1}{N_\alpha} \sum_{j=1}^{N_\alpha} e^{i\vec{q} \cdot (\vec{r}_j(t_w+t) - \vec{r}_j(t_w))} \right\rangle$$



- ▶ three time windows:
 - ▶ too short
 - ▶ intermediate (scaling)
 - ▶ equilibrium (t_{eq}^C)
- ▶ equilibrium curve included in scaling

Mean Square Displacement

$$\Delta r^2(t_w, t_w + t) = \left\langle \frac{1}{N} \sum_{j=1}^N (\mathbf{r}_j(t_w + t) - \mathbf{r}_j(t_w))^2 \right\rangle$$

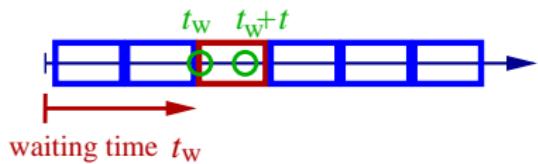
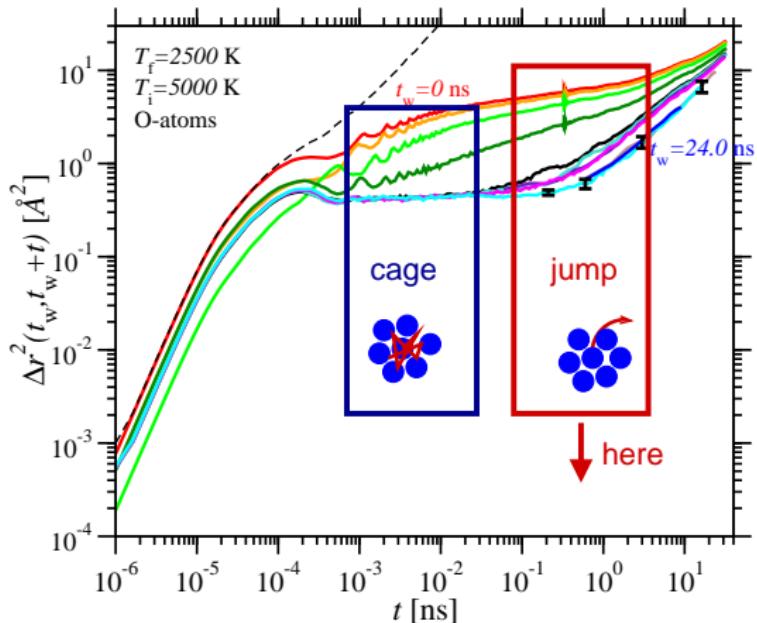


► three time windows:

[KVL, J. Roman, J.Horbach, PRE 81, 061203 (2010)]

Mean Square Displacement

$$\Delta r^2(t_w, t_w + t) = \left\langle \frac{1}{N} \sum_{j=1}^N (\mathbf{r}_j(t_w + t) - \mathbf{r}_j(t_w))^2 \right\rangle$$



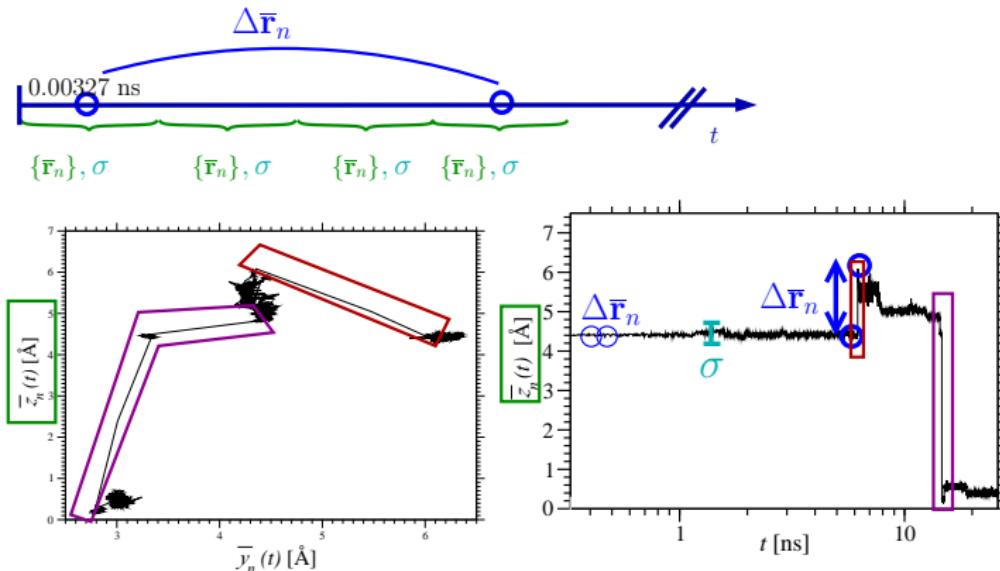
- ▶ three time windows:

Goal:

Single Particle Picture

(not $\frac{1}{N} \sum_{j=1}^N$)

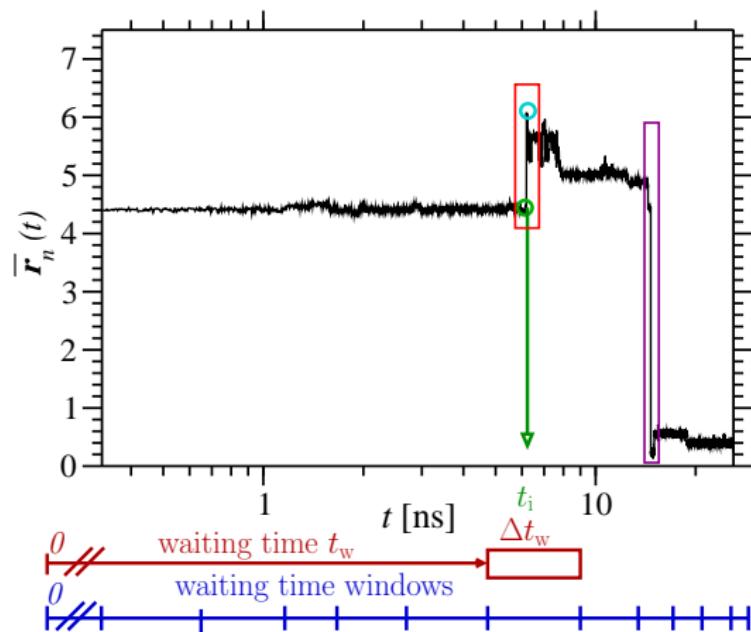
Jump Definition



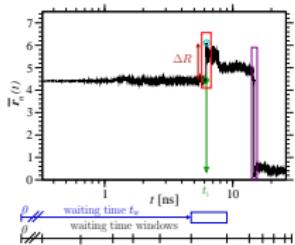
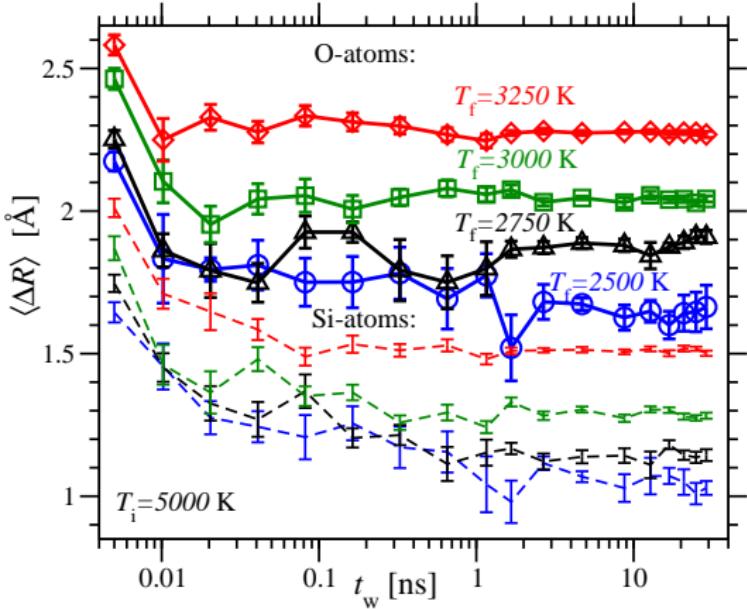
$$\Delta \bar{r}_n > 3 \sigma$$

[KVL, R. Bjorkquist, L.M. Chambers, PRL 110, 017801 (2013)]

Jump Definition: Aging Dependence

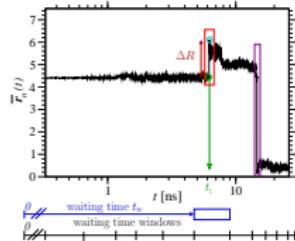
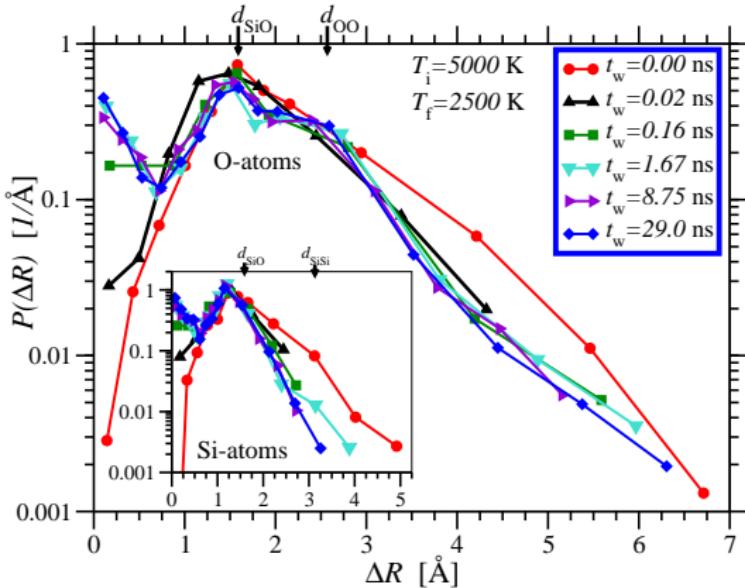


Average Jump Length



- ▶ O-atoms jump farther than Si-atoms
- ▶ compare:
 $d_{\text{SiO}} = 1.59 \text{ \AA}$, $d_{\text{OO}} = 2.57 \text{ \AA}$,
 $d_{\text{SiSi}} = 3.13 \text{ \AA}$
- ▶ ΔR mostly independent of t_w

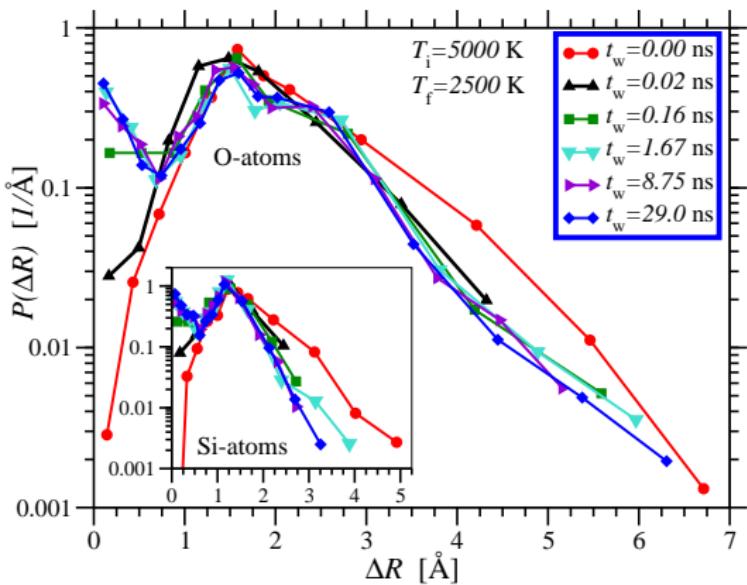
Jump Length Distribution



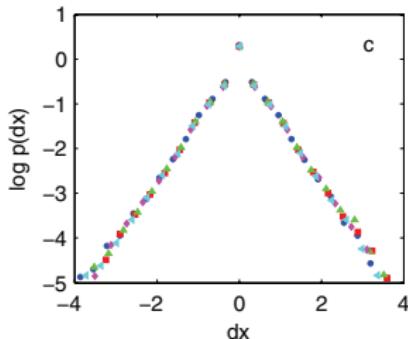
- ▶ peak at $\Delta R_j = 0$: reversible jumps
- ▶ peaks at d_{SiO} and d_{OO}
- ▶ exponential decay
- ▶ $P(\Delta R)$ independent of t_w

Jump Length Distribution

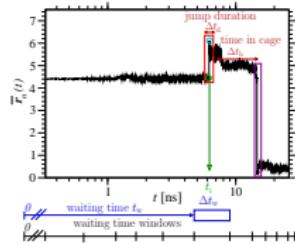
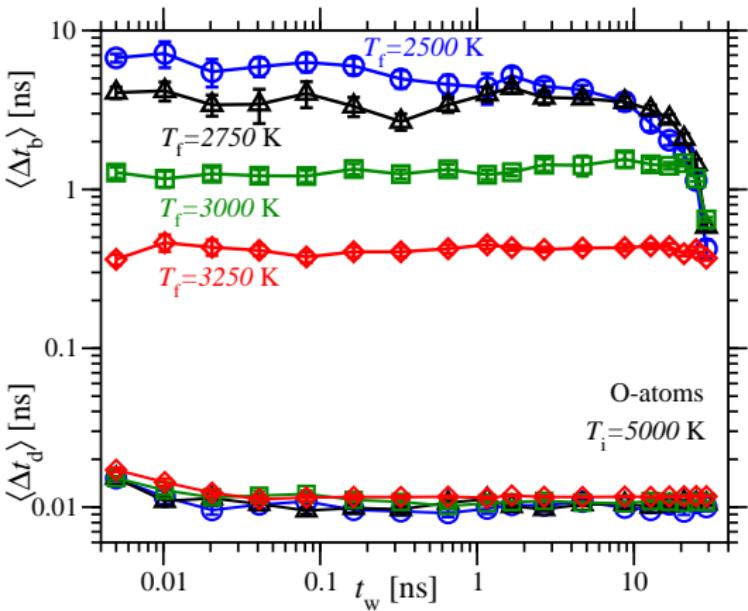
strong glass former SiO_2 :



- ▶ $P(\Delta R)$ independent of t_w
- ▶ exponential decay
- ▶ compare fragile glassformer binary LJ (& polymer)
[Warren & Rottler, EPL(2009)]

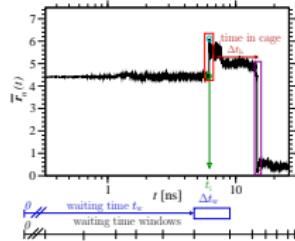
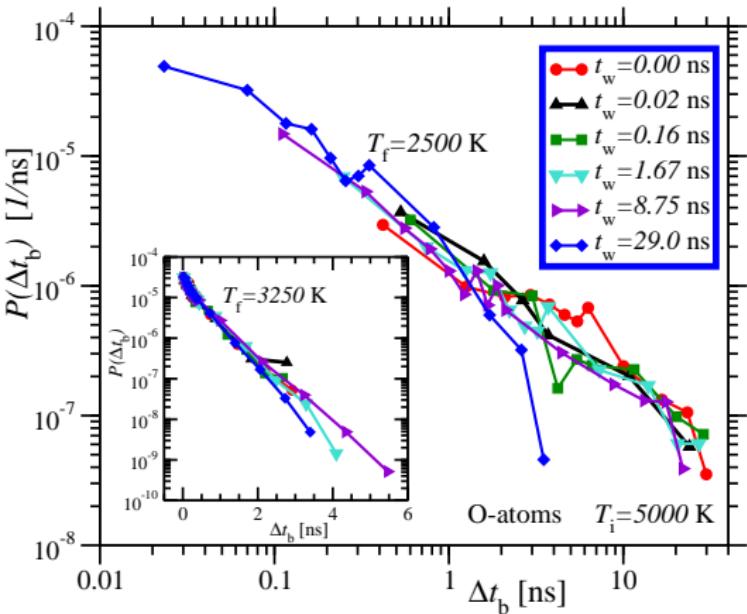


Time Averages: Jump Duration Δt_d & Time in Cage Δt_b



- ▶ $\Delta t_b \gg \Delta t_d$
- ▶ Δt_b not influenced by Δt_w
- ▶ $t_w \gtrsim 10$ artifact due to finite simulation run
- ▶ Δt_b independent of t_w !

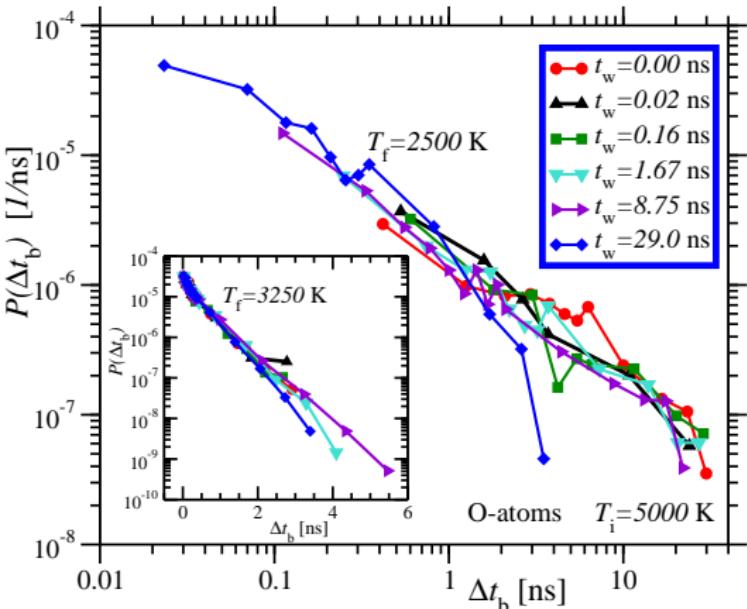
Distribution of Time in Cage $P(\Delta t_b)$



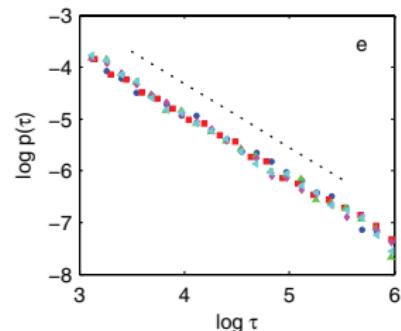
- ▶ $P(\Delta t_b)$ independent of t_w !
- ▶ 2500 K: powerlaw
- ▶ 3250 K: exponential

Distribution of Time in Cage $P(\Delta t_b)$

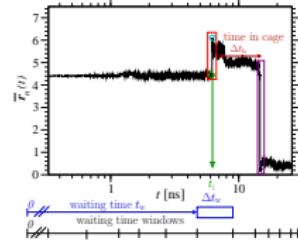
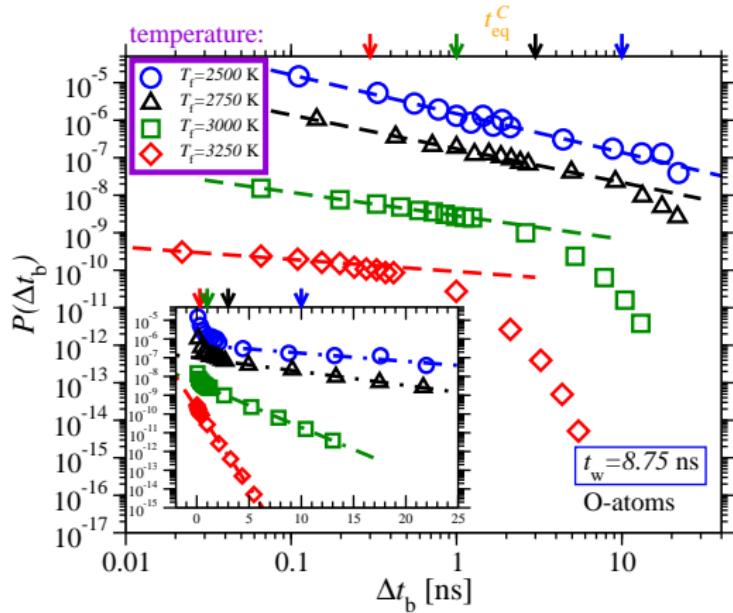
strong glass former SiO_2 :



- ▶ $P(\Delta t_b)$ independent of t_w !
- ▶ compare fragile glassformer
(binary LJ &) polymer
[Warren & Rrottler, EPL(2009)]



Distribution of Time in Cage $P(\Delta t_b)$: T_f varied

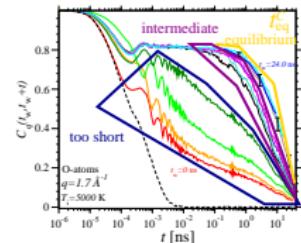
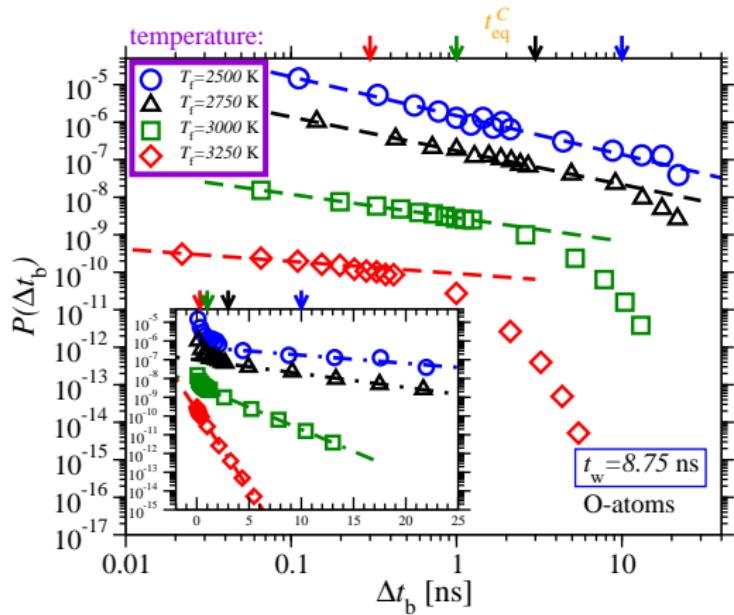


$t_w = 8.75 \text{ ns}$ fixed
temperature T_f varied

► crossover

- power law to exponential

Distribution of Time in Cage $P(\Delta t_b)$: T_f varied



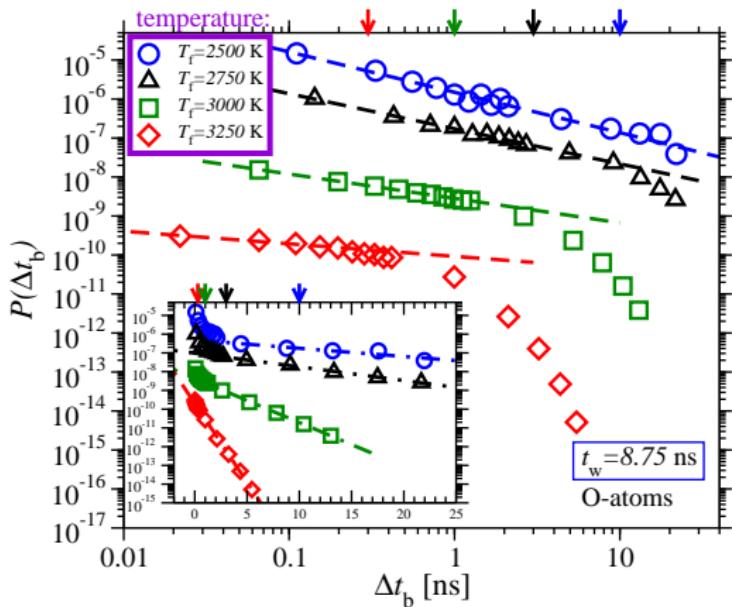
$t_w = 8.75 \text{ ns}$ fixed
temperature T_f varied

► crossover

- power law to exponential
- at $t_{\text{cross}} \approx t_{\text{eq}}^C$ (arrows)

Distribution of Time in Cage $P(\Delta t_b)$: T_f varied

strong glass former SiO_2 :



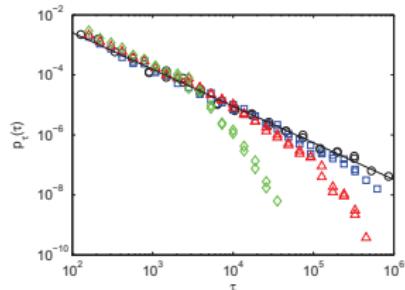
► crossover

- power law to exponential
- slope power law: -1.0 to -0.3 ($T_f = 2500 \text{ K}$ to $T_f = 3250 \text{ K}$)
- at $t_{\text{cross}} \approx t_{\text{eq}}^C$

► compare fragile glassformer binary LJ

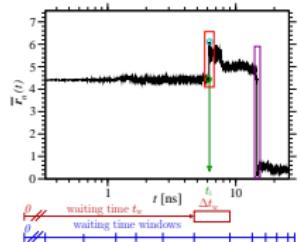
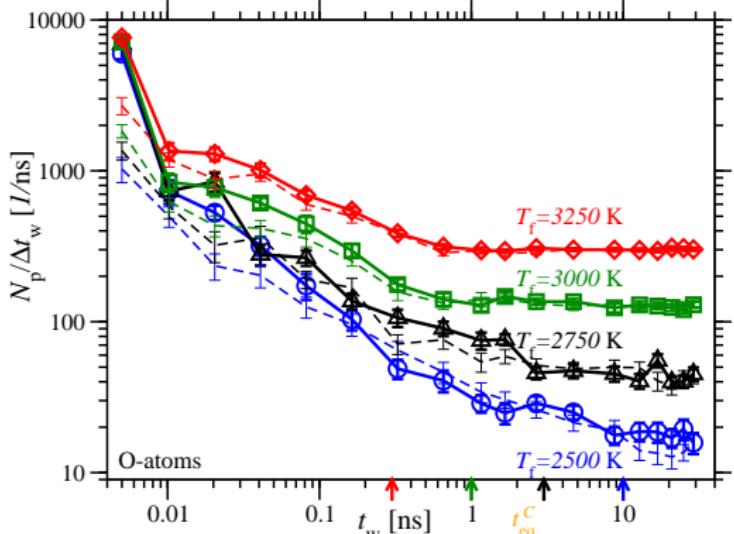
[Warren & Rottler, PRL (2013)]

- slope power law: -1.23 to -1.34 ($T_f = 0.28$ to $T_f = 0.37$)



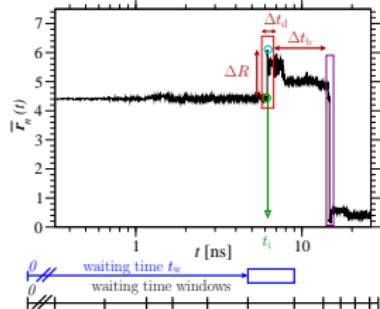
[KVL, R. Bjorkquist, L.M. Chambers, PRL (2013)]

Number of Jumping Particles per Time



- ▶ $N_p / \Delta t_w$ depends strongly on waiting time t_w
 - $N_p / \Delta t_{tw}$ decreasing with increasing t_w
 - compare: soft colloids [Yunker et al., PRL (2009)]
- ▶ equilibration at t_{eq}^j
 $t_{\text{eq}}^j \approx t_{\text{eq}}^C$ (arrows)

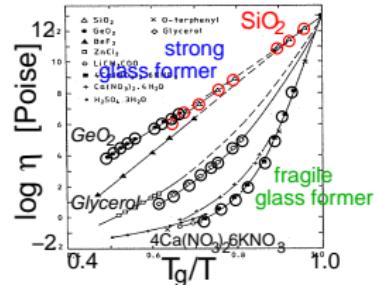
Summary: Microscopic Picture of Aging



Aging of SiO_2 :

- ▶ Only t_w -dependence: $N_p/\Delta t_w$ (not $P(\Delta R)$ and $P(\Delta t_b)$)
- ▶ $P(\Delta t_b)$ crossover power law to exponential
 - at $t_{\text{cross}} \approx t_{\text{eq}}^j \approx t_{\text{eq}}^C$

[KVL, R. Bjorkquist, L.M. Chambers, PRL 110, 017801 (2013)]



Compare with Fragile Glassformer:

- ▶ Surprising similar jump dynamics of strong and fragile glass formers
 - $P(\Delta R)$ and $P(\Delta t_b)$ t_w -independent
 - $P(\Delta t_b)$ crossover

PAST:

- ▶ Fragile Glass Former (Binary LJ): clusters of jumping particles
→ self-organized criticality
[KVL & Baker, EPL(2006)]
- ▶ granular fluid: simulation and hydrodynamic theory
[KVL, T. Aspelmeier, A. Zippelius, PRE 2010]

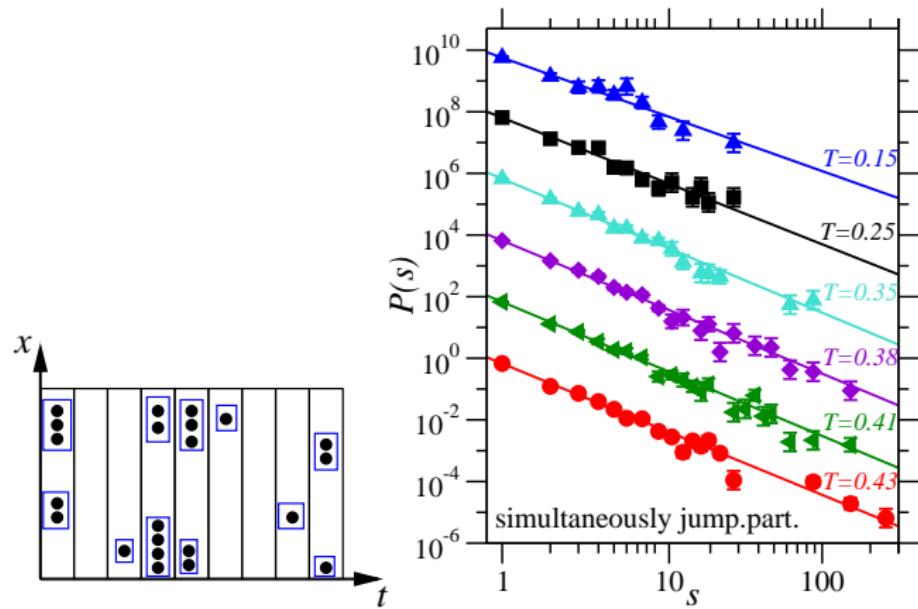
PRESENT:

Strong & Fragile Glass Former Similar?

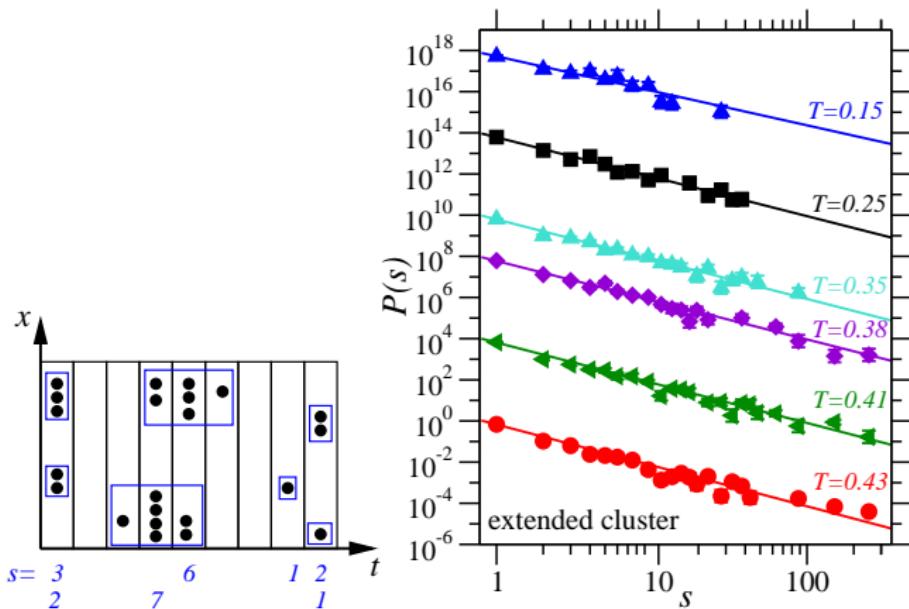
- ▶ SiO₂: scaling ($\chi_4, P(C_q)$)
together with H. Castillo
- ▶ SiO₂: defects & jumps
together with A. Zippelius

Acknowledgments: Supported by SFB 602, NSF REU grants PHY-0552790 & REU-0997424. Thanks to J. Horbach, A. Zippelius & University Göttingen.

Binary Lennard-Jones: Clusteranalysis (Simultaneous)



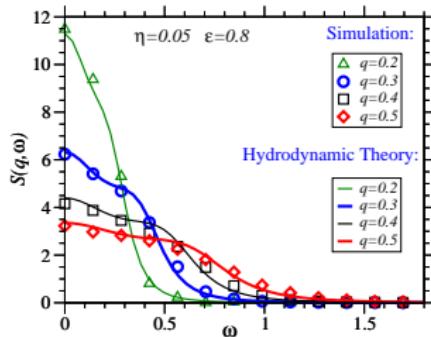
Binary Lennard-Jones: Clusteranalysis (Space-Time Cluster)



Summary of Granular Fluid Work

- Damped Sound Waves
- Fluctuating Hydrodynamic Theory:

- $D_T q^2 \approx \frac{3\Gamma_0}{2T_0}$ (full solution)
- $S(q, \omega)$ well approximated
- transport coefficients agree with kinetic theory



[KVL, T. Aspelmeier, A. Zippelius, PRE **83**, 011301 (2011)]

Theory: Fluctuating Hydrodynamics

$$\partial_t \delta n = -iqn_0 \mathbf{u}$$

$$\partial_t \mathbf{u} = -\frac{iq}{\rho_0} \left(\frac{\partial p}{\partial n} \delta n + \frac{\partial p}{\partial T} \delta T \right) - \nu_l q^2 \mathbf{u} + \xi_l$$

$$\partial_t \delta T = -D_T q^2 \delta T - \frac{3\Gamma_0}{2T_0} \delta T - iq \frac{2p_0}{dn_0} \mathbf{u} - \Gamma_0 \left(\frac{1}{n_0} + \frac{1}{\chi} \frac{d\chi}{dn} \right) \delta n + \theta$$

fluctuating number density $\delta n(\vec{q}, t) = n - n_0$

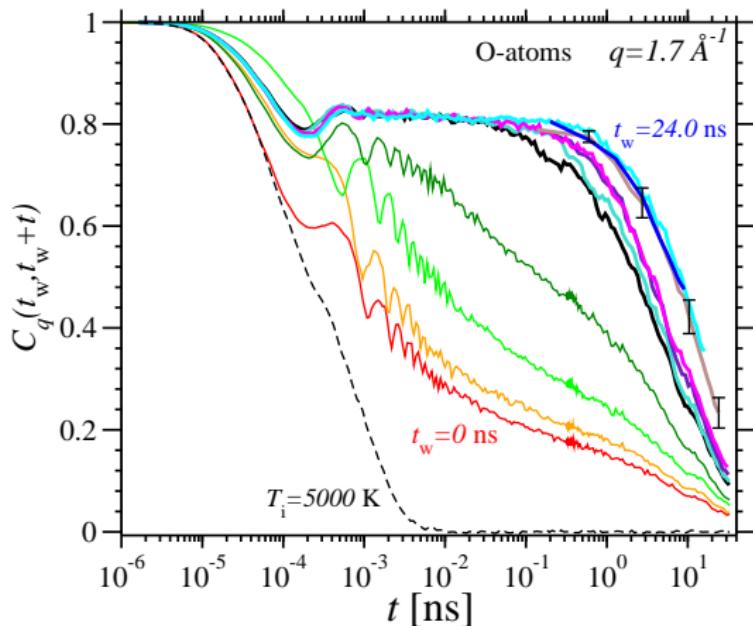
longitudinal flow velocity $u(\vec{q}, t) = \vec{u} \cdot \frac{\vec{q}}{q}$

fluctuating temperature $\delta T = T - T_0$

[Noije et al., PRE **59**, 4326 (1999)]

Generalized Intermediate Incoherent Scattering Function

$$C_q(t_w, t_w + t) = \left\langle \frac{1}{N_\alpha} \sum_{j=1}^{N_\alpha} e^{i\vec{q} \cdot (\vec{r}_j(t_w+t) - \vec{r}_j(t_w))} \right\rangle$$

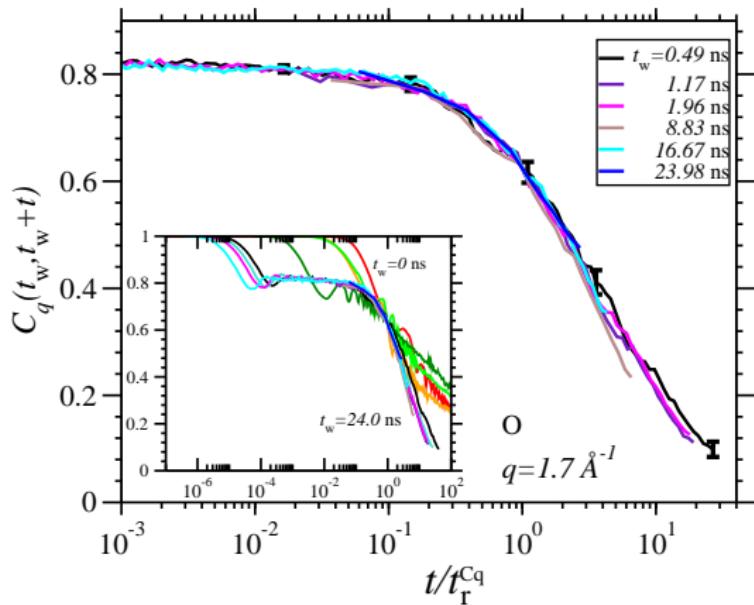


- ▶ t_w **small:**
 - $t_w = 0 \text{ & } t \lesssim 5 \cdot 10^{-5} \text{ ns}$:
 T_i good approx.
 - no plateau
 - decay t_w -dependent
- ▶ t_w **intermediate:**
 - plateau t_w -indep.
 - decay t_w -dependent
 - **time superposition ?**
- ▶ t_w **large:** t_w -indep.
→ **equilibrium**

Generalized Intermediate Incoherent Scattering Function

$$\text{MF: } C_q(t_w, t_w + t) = C_q^{\text{ST}}(t) + C_q^{\text{AG}} \left(\frac{h(t_w+t)}{h(t_w)} \right)$$

$$\text{Superposition: } C_q(t_w, t_w + t) = C_q^{\text{ST}}(t) + C_q^{\text{AG}} \left(\frac{t}{t_r^{Cq}(t_w)} \right)$$



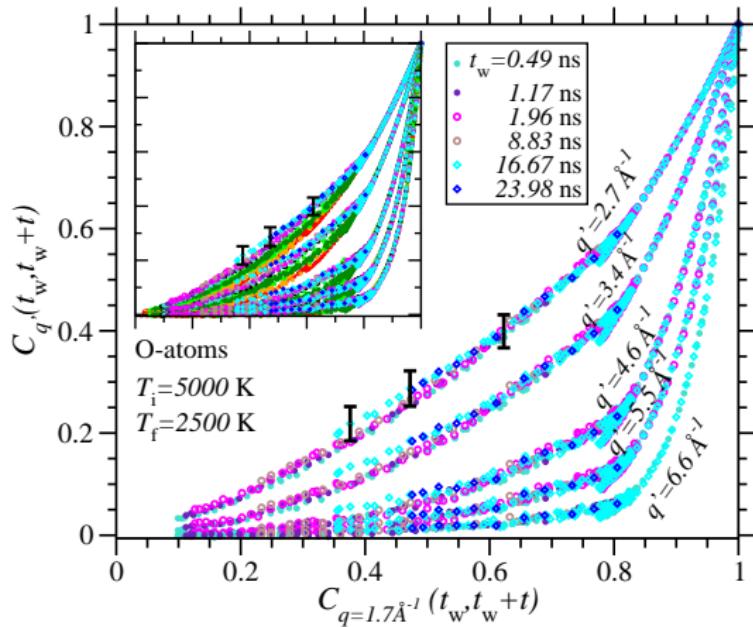
- ▶ t_w **small:**
no time
superposition
- ▶ t_w **intermediate:**
time superposition
- ▶ t_w **large:**
superposition
includes equilibrium
curve

LJ: [Kob & Barrat, PRL 78, 24 (1997)]

Generalized Intermediate Incoherent Scattering Function

$$C_q(t_w, t_w + t) = C_q^{\text{ST}}(t) + C_q^{\text{AG}} \left(\frac{h(t_w+t)}{h(t_w)} \right)$$

Is h dependent on C_q ?



- ▶ t_w **small:**
no superposition
- ▶ t_w **intermediate:**
superposition of
 $C_{q'}(C_q)$
 $\Rightarrow h$ indep. of C_q
- ▶ t_w **large:**
superposition
includes equilibrium
curve

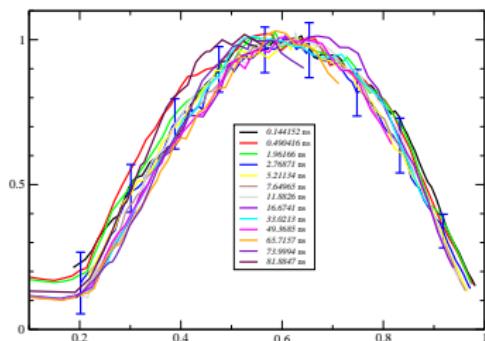
Dynamic Susceptibility

$$\chi_4(t_w, t_w + t) = N_\alpha \left[\left\langle (f_s(t_w, t_w + t))^2 \right\rangle - \langle f_s(t_w, t_w + t) \rangle^2 \right]$$

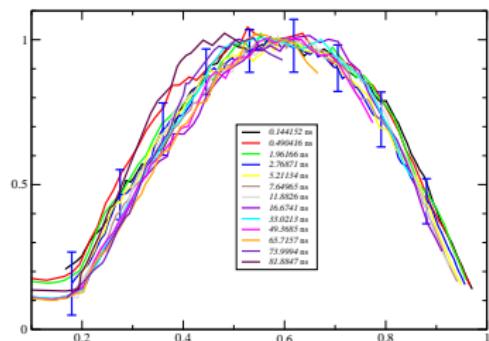
$$f_s(t_w, t_w + t) = \frac{1}{N_\alpha} \sum_{j=1}^{N_\alpha} e^{i \vec{q} \cdot (\vec{r}_j(t_w + t) - \vec{r}_j(t_w))}$$

$$C_q(t_w, t_w + t) = \langle f_s \rangle$$

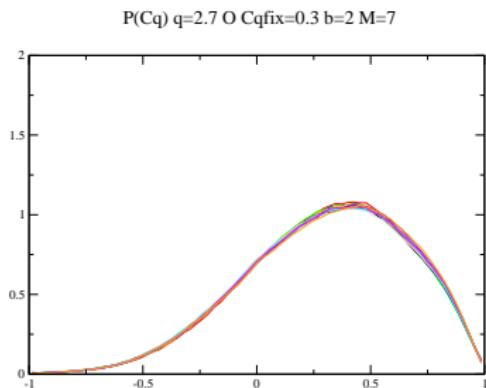
$$\chi_d^{Fs} / \chi_d^{max}(I-Fs) q=1.7 O$$



$$\chi_d^{Fs} / \chi_d^{max}(I-Fs) q=1.7 SiO$$



Local Incoherent Intermediate Scattering Function



Incoherent Intermediate Scattering Function

