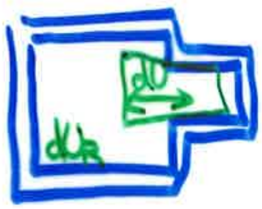
SUMMARY FOR EXAM 3

$$\overline{X} = \sum_s X(s)\,P(s)$$

Canonical Distribution

$$P(s) = \frac{e^{-E(s)/kT}}{Z}$$

$$Z = \sum_s e^{-E(s)/kT}$$

Derive: 

& apply

paramagnet, rotations, equipartition thm, composite systems, 3 state etc.

$Z \to F = -kT\ln Z \to \mu$ & S

$\searrow U = \overline{E} = -\frac{\partial}{\partial\beta}\ln Z$ ideal gas

Ising Model (FM)

in 1 dim. $Z \to U$ $(B=0)$

MF: derive & sketch $(B=0\ \&\ B\neq 0)$

Grand Canonical Distribution

derive

apply: CO & O_2 with hemoglobin

Bosons & Fermions: microstates

derive distributions

degenerate Fermi gas

BOLTZMANN STATISTICS I

2 state system

s	M	E
+1	$+\mu$	$-\mu B$
-1	$-\mu$	$+\mu B$

a) $\bar{M} = ?$

$$\bar{M} = \frac{\mu e^{-(-\mu B)/kT} - \mu e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \mu \tanh\left(\frac{\mu B}{kT}\right)$$

b) show $\chi = \frac{\partial \bar{M}}{\partial B} = \frac{1}{kT}\left(\overline{M^2} - \bar{M}^2\right)$

$$\chi = \frac{\partial \bar{M}}{\partial B} = -\frac{1}{Z^2}\frac{\partial Z}{\partial B}\left(\mu e^{\mu B/kT} - \mu e^{-\mu B/kT}\right) + \frac{\mu\left(\frac{\mu}{kT}\right)e^{\mu B/kT} - \mu\left(-\frac{\mu}{kT}\right)e^{-\mu B/kT}}{Z}$$

$$= -\frac{1}{Z^2}\left(\frac{\mu}{kT}e^{\mu B/kT} - \frac{\mu}{kT}e^{-\mu B/kT}\right)\left(\mu e^{\mu B/kT} - \mu e^{-\mu B/kT}\right) + \frac{\overline{M^2}}{kT}$$

$$= \frac{1}{kT}\left(\overline{M^2} - \bar{M}^2\right) \qquad \square$$

BOLTZMANN STATISTICS II

II Z for paramagnet $\quad e^{\mu B/kT} + e^{-\mu B/kT}$

$F = ?\qquad S = ?$

$$F = -kT\ln Z$$

$$F = -kT\ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right)$$

$$dF = -SdT - pdV + \mu dN$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = +k\ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right) + kT\,\frac{2\sinh\left(\frac{\mu B}{kT}\right)\left(-\frac{\mu B}{kT^2}\right)}{2\cosh\left(\frac{\mu B}{kT}\right)}$$

$$= k\ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right) - \frac{\mu B}{T}\tanh\left(\frac{\mu B}{kT}\right)$$

BOLTZMANN STATISTICS III

Single particle 3dim: $E = \frac{1}{2}mv^2 + \frac{k}{2}r^2$

a) $Z = ?$ show $Z_1 \cdot Z_2$ (don't try to get number)

$$Z = \int dv_x \int dv_y \int dv_z \int dx \int dy \int dz \; e^{-E/kT}$$

$$= \left(\int_0^\infty 4\pi v^2 e^{-\frac{mv^2}{2kT}} dv\right)\left(\int_0^\infty 4\pi r^2 e^{-\frac{k}{2}r^2/kT} dr\right)$$

b) $\bar{v} = ?$

$$\bar{v} = \frac{1}{Z_1}\int_0^\infty 4\pi v^3 e^{-\frac{mv^2}{2kT}} dv$$

because r^2 part cancels out

IV Gibbs Statistics I

Derive the Fermi-Dirac Distribution

$$P(s) = \frac{1}{Z} e^{-(E(s) - \mu N(s))/kT}$$

n particles in particle states

$$P(s) = \frac{1}{Z} e^{-(n\varepsilon - n\mu)/kT}$$

Fermions : $n = 0, 1$

$$\bar{n}_{FD} = \frac{\sum_{n=0,1} n\, e^{-(n\varepsilon - n\mu)/kT}}{\sum_{n=0,1} e^{-(n\varepsilon - n\mu)/kT}} = \frac{0 \cdot e^{0} + 1 \cdot e^{-(\varepsilon-\mu)/kT}}{e^{0} + e^{-(\varepsilon-\mu)/kT}}$$

$$= \frac{e^{-(\varepsilon-\mu)/kT}}{1 + e^{-(\varepsilon-\mu)/kT}} = \frac{1}{e^{(\varepsilon-\mu)/kT} + 1}$$

Ising FM I & II

For the Ising ferromagnet with MF approx.
for single spin energy:

$$E_{\uparrow} = -\varepsilon n \bar{s} - \mu_B B \qquad E_{\downarrow} = \varepsilon n \bar{s} + \mu_B B$$

$$s_{i\uparrow} = +1 \qquad s_{i\downarrow} = -1$$

a) Derive the self-consistent equation for $\bar{s} = \bar{s}_i$

$$\bar{s} = \bar{s}_i = \frac{\sum_{s_i} s_i e^{-E(s_i)/kT}}{\sum_{s_i} e^{-E(s_i)/kT}} = \frac{e^{[(\varepsilon n \bar{s} + \mu_B B)/kT]} - e^{-[\dots]}}{e^{[\dots]} + e^{-[\dots]}}$$

$$\bar{s} = \tanh\left[\beta \varepsilon n \bar{s} + \beta \mu_B B\right]$$

b) Sketch the solution(s) B small and T small