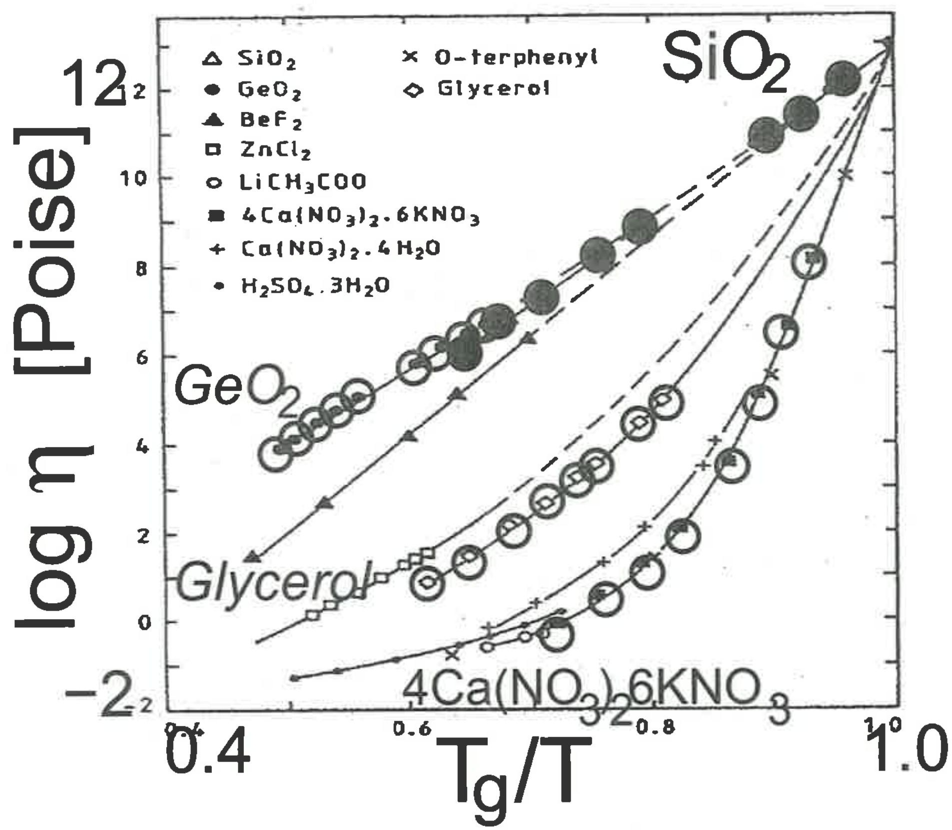
Introduction: Glass



Dynamics:

Viscocity η as function of inverse temperature T

- slowing down of many decades
  $\longrightarrow$ very interesting dynamics

[C.A. Angell and W. Sichina, Ann. NY Acad. Sci. 279, 53 (1976)]

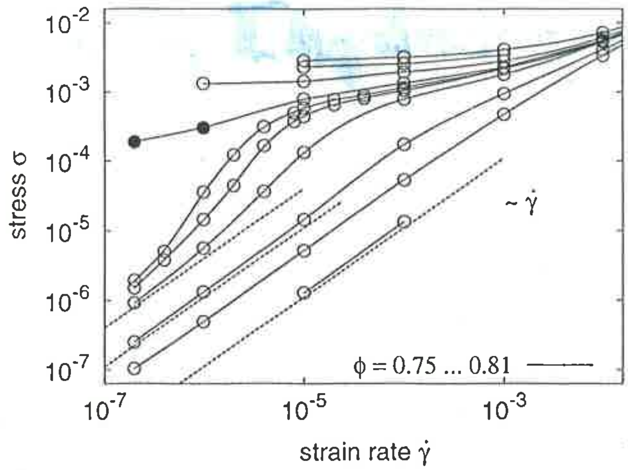


FIG. 1. Flow curves $\sigma(\dot\gamma)$ for various volume fractions $\phi = 0.75$, 0.77, 0.78, 0.79, 0.7925, 0.7935, 0.795, 0.8, 0.805, 0.81 (from bottom to top).

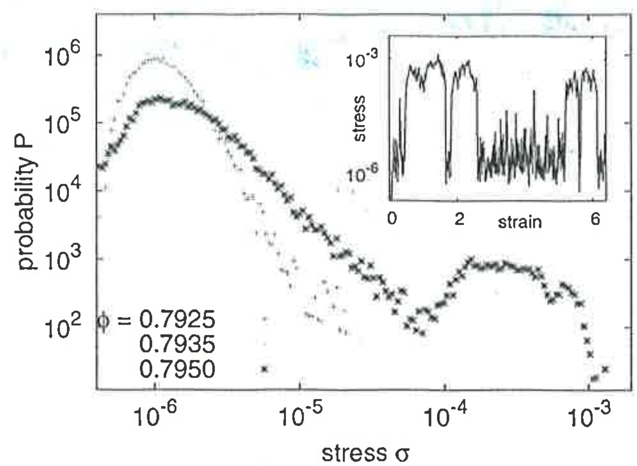


FIG. 3. (Color online) Probability distribution of stress values for different volume fractions ϕ and for $\dot\gamma = 2 \times 10^{-7}$. The double-peak structure (for $\phi = 0.795$) indicates the coexistence of jammed and viscous flow regimes. Inset: Stress-strain relation in the coexisting state.

zero stress, $\sigma_c = 0$. We will show in the following how a simple change to finite and constant friction coefficient $\mu \neq 0$ can fundamentally change this picture.

Results. In Figs. 1 and 2 we display the flow curves and the associated viscosities of our frictional simulations. By varying the volume fraction we go through the jamming transition and observe the associated changes in the flow behavior. At small volume fractions, below the jamming transition, we observe a Newtonian regime $\sigma = \eta_0\dot\gamma$, with a strain-rate-independent viscosity $\eta_0(\phi)$ that increases with volume fraction. At high densities, above jamming, the stress levels off at the yield stress, $\sigma_y(\phi) = \sigma(\dot\gamma \to 0,\phi)$.

In frictionless systems the jamming transition is associated with "critical" shear thinning $\sigma \sim \dot\gamma^x$ ($x < 1$, power-law fluid) [11,12,14]. Here, surprisingly, the opposite is happening: Jamming is signalled by a shear-thickening regime that grows stronger with increasing the volume fraction. At $\phi = 0.78$ only a mild increase of the viscosity is observed, before it drops in the shear-thinning regime. At $\phi = 0.7935$ the viscosity already increases by about an order of magnitude.

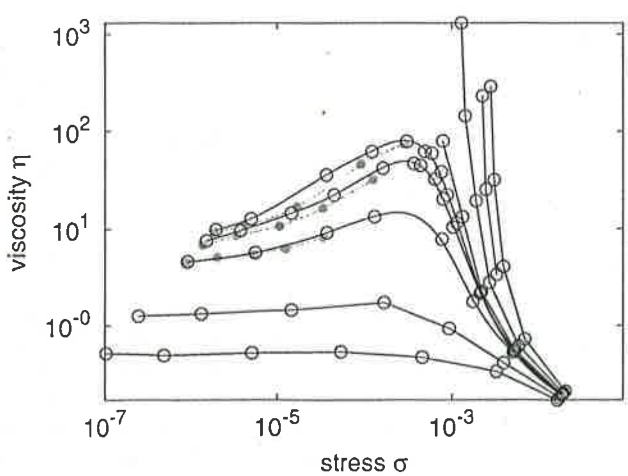


FIG. 2. (Color online) Viscosity $\eta = \sigma/\dot\gamma$ vs stress σ for various volume fractions $\phi = 0.77 \ldots 0.81$ ($N = 4900$). As a comparison the data from the $N = 10\,000$ system are given with small (red) symbols.

The stress scale in the thickening regime (as characterized, for example, by the stress at the viscosity maximum) is nearly independent of volume fraction. By way of contrast, the strain rate for the onset of thickening decreases with volume fraction (the thickening regime shifts to the left in Fig. 1). This shift does not go down to $\dot\gamma \to 0$. Rather, at about $\phi = 0.795$, the solid data points in Fig. 1 indicate qualitatively different behavior: the *coexistence* of jammed solid and freely flowing fluid states. This is evidenced in Fig. 3. For the solid data points the stress distribution is bimodal (black star) and the stress-strain relation shows sudden switching events from low-stress (fluid) to high-stress (solid) states. By way of contrast, in the (continuous) thickening regime (red plus, green cross) the stress distributions have only one peak. As can be seen in the figure, the tails of this distribution are rather broad, indicative of giant stress fluctuations.

Discussion. The observed phenomena are strongly reminiscent of critical behavior. The coexistence of flowing and jammed states then signals a discontinuous jamming transition (similar to the dry granular flow of Ref. [18]). The coexistence region seems to be terminated by a "critical point" at a certain (nonzero) value of stress, an associated strain rate, and a volume fraction ($\sigma_c,\dot\gamma_c,\phi_c$), at which the transition is continuous. The shear-thickening regime then corresponds to the near-critical "isochores" close to but above this point.

Evidence of this scenario of a finite-stress critical point is provided by the fact that stress fluctuations in the shear-thickening regime are strongly enhanced. Equally important, a large correlation length indicates cooperative behavior. To extract such a length scale we calculate the velocity correlation function $C_v(x) = \langle v_y(x)v_y(0)\rangle$, where we concentrate on the velocity component in the gradient direction v_y of two particles separated by x in the flow direction. In the frictionless system this correlation function has been used to evidence a correlation

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We expand in fluctuations around the stationary state: $n = n_0 + \delta n, T = T_0 + \delta T$ and $\Gamma = \Gamma_0 + \delta\Gamma$. The collision frequency should be proportional to the density, the pair correlation function at contact, χ, and the thermal velocity: $\nu_{\text{coll}} \propto n\chi T^{1/2}$, hence linearization around the stationary state Γ_0 yields: $\Gamma \sim \Gamma_0(1+\frac{\delta n}{n_0}+\frac{1}{\chi}\frac{\mathrm{d}\chi}{\mathrm{d}n}\delta n+\frac{3\delta T}{2T_0})$.

Following van Noije et al. [19], we consider a hydrodynamic description of a granular fluid based on conservation of particle number and momentum and the relaxation of temperature to its stationary value, T_0. The transverse momentum decouples so that we are left with three equations for the fluctuating density δn, the longitudinal flow velocity $u(\mathbf{q},t) = \mathbf{q}\cdot\mathbf{u}/q$, and the fluctuating temperature δT:

$$\partial_t \delta n(\mathbf{q},t) = -iqn_0 u(\mathbf{q},t) \tag{15}$$

$$\begin{aligned}\partial_t u(\mathbf{q},t) &= -\frac{iq}{mn_0}\left(\frac{\partial p}{\partial n}\delta n(\mathbf{q},t) + \frac{\partial p}{\partial T}\delta T(\mathbf{q},t)\right) \\ &- \nu_l q^2 u(\mathbf{q},t) + \xi_l(\mathbf{q},t)\end{aligned} \tag{16}$$

$$\begin{aligned}\partial_t \delta T(\mathbf{q},t) &= -D_T q^2 \delta T(\mathbf{q},t) - iq\frac{2p_0}{dn_0}u(\mathbf{q},t) \\ &- \Gamma_0\left(\frac{\delta n(\mathbf{q},t)}{n_0} + \frac{1}{\chi}\frac{\mathrm{d}\chi}{\mathrm{d}n}\delta n(\mathbf{q},t) + \frac{3}{2}\frac{\delta T(\mathbf{q},t)}{T_0}\right) \\ &+ \theta(\mathbf{q},t),\end{aligned} \tag{17}$$

where $D_T = \frac{2\kappa}{dn_0}$ with the heat conductivity κ, and where ν_l is the longitudinal viscosity. Fluctuating hydrodynamics for an elastic fluid ($\epsilon = 1$) is based on internal noise, ξ_l^{in} and θ^{in}, consistent with the fluctuations-dissipation