PHYS 317 Fall 2018

Summary for Exam 1

Ideal Gas:

$pV = NkT = \nu RT$ and microscopic picture

Equipartition Theorem:

$U_{\text{therm}} = N\frac{f}{2}kT$ (apply and determine f)

1st Law of Thermodynamics:

$\Delta U = Q + W$ $W = -\int p\mathrm{d}V$

(pV diagrams, adiabat, isotherm, straight lines)

Heat Capacities and Enthalpy:

$C = \frac{Q}{\Delta T}$ $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ $C_p = \left(\frac{\partial H}{\partial T}\right)_p$

$C = m\,c$

$H = U + pV$ (apply to reactions; if on exam, then table will be provided)

Heat Conduction, Diffusion: microscopic picture

Multiplicities:

systems: 2-state, Einstein solid, ideal gas (& similar)

derive Ω, Ω_{tot}, apply Stirling formula and $\ln(1+x) \approx x$, know EXCEL commands

Entropy: $S = k\,\ln\Omega$

determine S, ΔS

2nd Law of Thermodynamics: major concept

Formulae for Exam #1

$$k = 1.381 \cdot 10^{-23} \text{ J/K} = 8.617 \cdot 10^{-5} \text{ eV/K}$$

$$h = 6.626 \cdot 10^{-34} \text{ Js} = 4.136 \cdot 10^{-15} \text{ eVs}$$

$$N_\text{A} = 6.022 \cdot 10^{23}$$

$$\Delta U = Q + W_\text{on} \qquad \text{where} \qquad W_\text{on} = -\int p \mathrm{d}V$$

$$C = \frac{Q}{\Delta T}$$

$$pV = NkT \qquad pV^\gamma = \text{const., where } \gamma = (f+2)/f$$

$$U = \frac{f}{2}NkT \qquad \frac{1}{2}kT \text{ for each quadratic degree of freedom}$$

$$H = U + pV$$

$$\Omega = \binom{q+N-1}{q} \qquad \Omega = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

$$\ln N! \approx N \ln N - N \qquad \ln(1+x) \approx x$$

$$S = k \ln \Omega$$

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

Ideal Gas: (1 J)

For an ideal gas you change

$p \to 3p \qquad N \to 4N \qquad V \to 8V$

Question: How does T change? $T \to ?$

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Question: How does T change? $T \to ?$

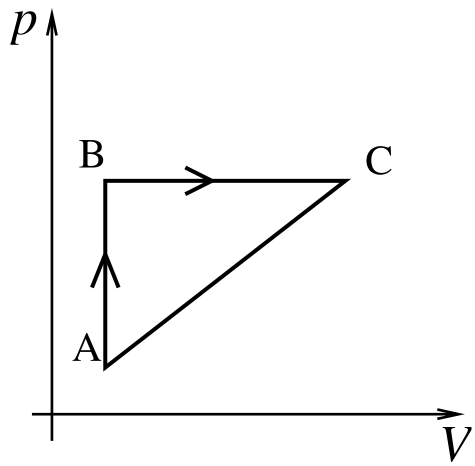
Answer:

$pV = NkT \longrightarrow T = \frac{pV}{Nk}$

$T \to \frac{(3p)(8V)}{(4N)k} = 6\frac{pV}{Nk} = 6T$

Ideal Gas: (2 J)

An ideal diatomic gas is made to undergo the following process:



Question: For this process fill all empty entries in the following table with +,-, or 0 depending on the sign of each quantity.

	W	Q	ΔU
$A \to B$			
$B \to C$			
$A \to B \to C \to A$			

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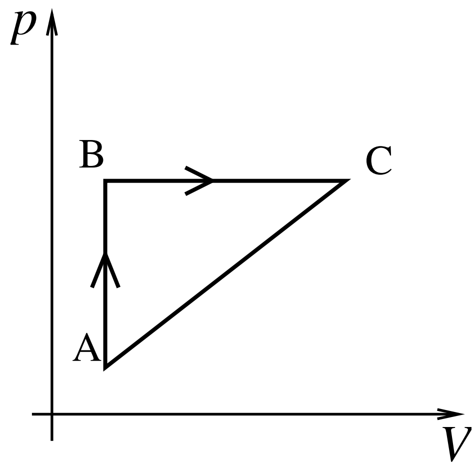

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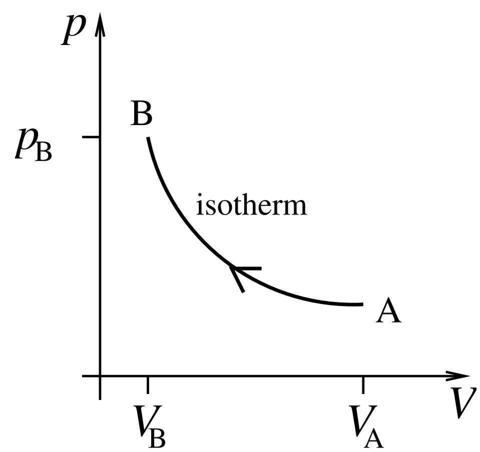
Answer:

	W	Q	ΔU
$A \to B$	0	+	+
$B \to C$	-	+	+
$A \to B \to C \to A$	-	+	0

Ideal Gas: (3 J)

An ideal diatomic gas at very high temperature is compressed along an isotherm as sketched in the figure below.



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$\Delta U, Q$, and W.

Express your results in terms of p_B, V_A, and V_B.

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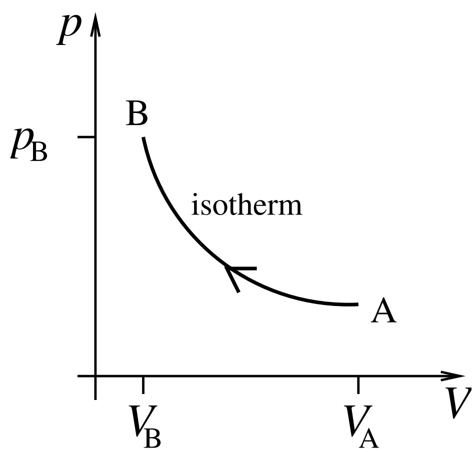

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Answer:

$$W = -\int p \mathrm{d}V = -\int \frac{NkT}{V} \mathrm{d}V = -NkT \int \frac{1}{V} \mathrm{d}V = -NkT \ln V \Big|_{V_A}^{V_B}$$

$$W = -p_B V_B \ln\left(\frac{V_B}{V_A}\right)$$

$$\Delta U = \frac{f}{2} Nk\Delta T = 0$$

$$\Delta U = Q + W \quad \longrightarrow \quad Q = -W = p_B V_B \ln\left(\frac{V_B}{V_A}\right)$$

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What is Ω for an Einstein solid, $\Omega(q, N)$?

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Answer:

$\Omega(q, N) = \begin{pmatrix} q + N - 1 \\ q \end{pmatrix} = \frac{(q+N-1)!}{q!(N-1)!} \approx \frac{(q+N)!}{q!N!}$

Mulplicities & Entropy: (2 J)

Question:

Approximate $\Omega(q, N) = \frac{(q+N)!}{q!N!}$ assuming that both q and N are very large, and that $q \gg N$ (that corresponds to large temperature. Do the derivation (even if you remember the result).

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Answer:

$$
\begin{aligned}
\ln \Omega(q, N) &\approx (q+N)\ln(q+N) - \cancel{(q+N)} - q\ln q + \cancel{q} - N\ln N + \cancel{N} \\
&= (q+N)\ln(q+N) - q\ln q - N\ln N \\
&= (q+N)\ln\left(q(1+\frac{N}{q})\right) - q\ln q - N\ln N \\
&= (q+N)\ln q + (q+N)\ln\left(1+\frac{N}{q}\right) - q\ln q - N\ln N \\
&\approx (q+N)\ln q + (q+N)\left(\frac{N}{q}\right) - q\ln q - N\ln N \\
&= q\ln q + N\ln q + N + \frac{N^2}{q} - q\ln q - N\ln N \\
&\approx \cancel{q\ln q} + N\ln q + N - \cancel{q\ln q} - N\ln N \\
&= N\ln\left(\frac{q}{N}\right) + N \\
\ln \Omega(q, N) &= \ln\left(\frac{q}{N}\right)^N + \ln\left(e^N\right)
\end{aligned}
$$

$$
\Omega = e^{\ln \Omega} \approx \left(\frac{qe}{N}\right)^N
$$

Mulplicities & Entropy: (2 J)

For the Einstein solid with $q, N \gg 1$ and the case $q \ll N$ (low temperature) the multiplicity is approximately

$$\Omega \approx \left(\frac{Ne}{q}\right)^q$$

Question:
Determine ΔS if you keep q fixed but increas N to $3N$.

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Determine ΔS if you keep q fixed but increas N to $3N$.

Answer:

$$\begin{aligned}
\Delta S = S(q, 3N) - S(q, N) &= k \ln \Omega(q, 3N) - k \ln \Omega(q, N) \\
&= kq \ln\left(\frac{3Ne}{q}\right) - kq \ln\left(\frac{Ne}{q}\right) \\
&= kq \ln\left(\frac{3Ne/q}{Ne/q}\right) = \boxed{kq \ln 3}
\end{aligned}$$

Equipartition Thm & Heat Capacity: (1 J)

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Determine the number of degrees of freeedom for SiO_2 at very high temperature.

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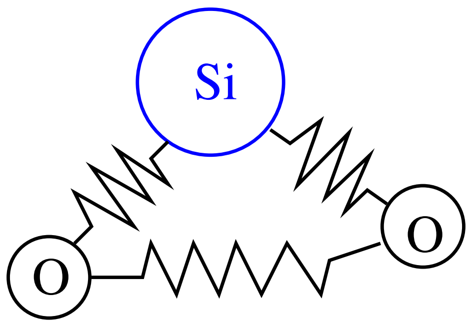

f= 3 translational + 3 rotational + 3 $\times$ 2 vibrational

$\boxed{f = 12}$

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Answer:

$$C_p = \left(\frac{Q}{\Delta T}\right)_p \overset{\text{1stLaw}}{=\!=} \left(\frac{\Delta U - W}{\Delta T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

$$\overset{\text{diatom.,highT}}{=} \frac{\partial}{\partial T}\left(N\frac{(3+2+2)}{2}kT\right)_p + p\frac{\partial}{\partial T}\left(\frac{NkT}{p}\right)_p$$

$$= \frac{7}{2}Nk + Nk = \boxed{\frac{9}{2}Nk}$$