Specific Heat, Entropy

(1 diamond) $C_V = aT + bT^3 + cT^4$

Determine ΔS for $T_i = 0$ to T_f

$$\Delta S = \int_0^{T_f} \frac{C_V dT}{T} = \int_0^{T_f} \frac{(aT + bT^3 + cT^4)}{T} dT = aT_f + \frac{b}{3}T_f^3 + \frac{c}{4}T_f^4$$

(2 diamonds) For a monatomic ideal gas $\Omega \to S = k\ln\Omega$

$$S = Nk\ln V + Nk\ln(U^{3/2}) + f(N)$$

Determine $T(U)$ and C_V

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} = \frac{\partial}{\partial U}\left(\frac{3}{2}Nk\ln U\right) = \frac{3}{2}\frac{Nk}{U}$$

$$T = \frac{2U}{3Nk}$$

$$U = \frac{3}{2}NkT$$

[sometimes e.g. $\frac{\partial S}{\partial U} = \frac{\partial S}{\partial N_r}\frac{\partial N_r}{\partial U}$ easier than $\frac{\partial S}{\partial U}$]

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}Nk$$

Thermodynamic Identities

$$\Psi = U + pV - \mu N$$

(1 diamond) What is thermodynamic identity $d\Psi$?

$$dU = TdS - pdV + \mu dN$$

$$d\psi = dU + pdV + Vdp - \mu dN - Nd\mu$$

$$= TdS - \cancel{pdV} + \cancel{\mu dN} + \cancel{pdV} + Vdp - \cancel{\mu dN} - Nd\mu$$

$$d\psi = TdS + Vdp - Nd\mu$$

(2 diamonds) Determine Maxwell relation between T and N.

$$\frac{\partial}{\partial \mu}\frac{\partial \psi}{\partial S} = \frac{\partial}{\partial S}\frac{\partial \psi}{\partial \mu}$$

$$\left(\frac{\partial T}{\partial \mu}\right)_{p,S} = -\left(\frac{\partial N}{\partial S}\right)_{p,\mu}$$

Phase Transitions

(1 diamond)

Which of the following statement(s) are true?

i) $pV = NkT$ is a model for phase transitions

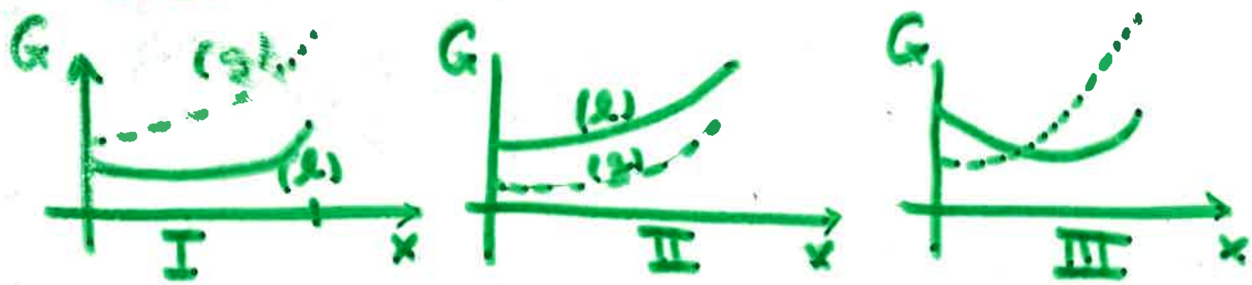
ii) at $V = V_c$ $\frac{\partial p}{\partial V} = 0$ & $\frac{\partial^2 p}{\partial V^2} = 0$

iii) a non ideal mixture has a phase transition

i) no if you wish at $V \to 0$ ii) yes iii) yes, characterized by phase seperation

(2 diamonds)

For the scenarios below map the G(x) to horizontal lines in T(x): Answer A, B, C or D.



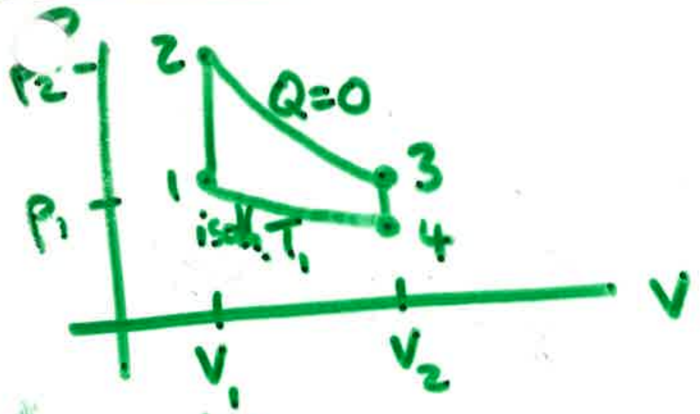
A: I II III

B: I II III

C: I II III

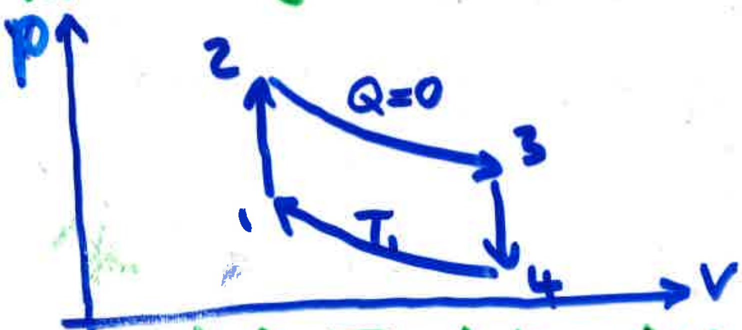
D: I II III

Heat Engines & Refridgerators



ideal gas
f degrees of freedom

(diamond) To make engine, in which direction go arrows?



(2 diamonds) To determine the efficiency $\varepsilon = \frac{W}{Q_h} = \frac{W_{by}}{Q_h}$

we need W   W = ?   fine to keep $T_1, T_2, \ldots$ $V_1, \ldots, P_1 \ldots$

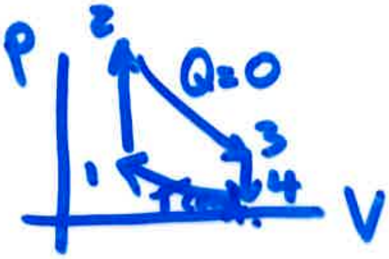
$$W = \underbrace{W_{12}}_{=0} + W_{23} + \underbrace{W_{34}}_{=0} + W_{41}$$

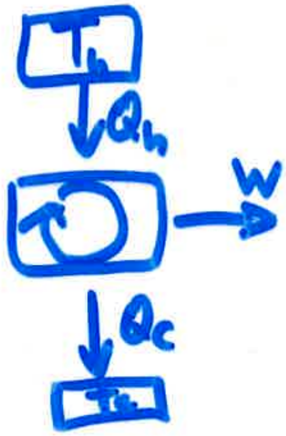
$$W_{23} = -\overbrace{\left[\Delta U_{23} - \underbrace{Q_{23}}_{=0}\right]}^{W_{on}} = -\frac{f}{2}Nk(T_3 - T_2) = \frac{f}{2}Nk(T_2 - T_3)$$

$$W_{41} = \int_4^1 p\,dV = \int_4^1 \frac{NkT_1}{V}\,dV = NkT_1 \ln\left(\frac{V_1}{V_4}\right)$$

$$W = \frac{f}{2}Nk(T_2 - T_3) + NkT_1 \ln\left(\frac{V_1}{V_4}\right)$$

(2 diamonds) Now get Q_h





$1 \rightarrow 2$: $W_{12} = 0$ $\quad \Delta U_{12} = \frac{f}{2} Nk(T_2 - T_1) = Q_{12} > 0$

↓ contributes to Q_h

$2 \rightarrow 3$: $Q = 0 \rightarrow$ no contribution to Q_h

$3 \rightarrow 4$: $W_{34} = 0$ $\quad \Delta U_{34} = Q_{34} < 0 \rightarrow$ no contribution to Q_h (contributes to Q_c)

$4 \rightarrow 1$: $\Delta U_{41} = \frac{f}{2} Nk(T_1 - T_4) = 0$

$\Delta U_{41} = Q_{41} + W_{41} \rightarrow Q_{41} = -W_{41} = \int_4^1 p\,dV < 0$

↓ contributes to Q_c

$Q_h = Q_{12} = \frac{f}{2} Nk(T_2 - T_1)$