BOLTZMANN STATISTICS I

2 state system

S	M	E
+1	$+\mu$	$-\mu B$
−1	$-\mu$	$+\mu B$

a) $\bar{M} = ?$

$$\bar{M} = \frac{\mu e^{-(-\mu B)/kT} - \mu e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \mu \tanh\left(\frac{\mu B}{kT}\right)$$

b) show $\chi = \frac{\partial \bar{M}}{\partial B} = \frac{1}{kT}\left(\overline{M^2} - \bar{M}^2\right)$

$$\chi = \frac{\partial \bar{M}}{\partial B} = -\frac{1}{Z^2}\frac{\partial Z}{\partial B}\left(\mu e^{\mu B/kT} - \mu e^{-\mu B/kT}\right) + \frac{\mu\left(\frac{\mu}{kT}\right)e^{\mu B/kT} - \mu\left(-\frac{\mu}{kT}\right)e^{-\mu B/kT}}{Z}$$

$$= -\frac{1}{Z^2}\left(\frac{\mu}{kT}e^{\mu B/kT} - \frac{\mu}{kT}e^{-\mu B/kT}\right)\left(\mu e^{\mu B/kT} - \mu e^{-\mu B/kT}\right) \quad \frac{\overline{M^2}}{kT}$$

$$= \frac{1}{kT}\left(\overline{M^2} - \bar{M}^2\right) \qquad \square$$

BOLTZMANN STATISTICS II

II Z for paramagnet $e^{\mu B/kT} + e^{-\mu B/kT}$

$F = ?$ $S = ?$

$$F = -kT \ln Z$$

$$F = -kT \ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right)$$

$$dF = -S\,dT - p\,dV + \mu\,dN$$

$$S \overset{\downarrow}{=} -\left(\frac{\partial F}{\partial T}\right)_{V,N} = +k\ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right) + kT\,\frac{2\sinh\left(\frac{\mu B}{kT}\right)\left(-\frac{\mu B}{kT^2}\right)}{2\cosh\left(\frac{\mu B}{kT}\right)}$$

$$= k\ln\left(2\cosh\left(\frac{\mu B}{kT}\right)\right) - \frac{\mu B}{T}\tanh\left(\frac{\mu B}{kT}\right)$$

BOLTZMANN STATISTICS III

Single particle 3 dim: $E = \frac{1}{2}mv^2 + \frac{k}{2}r^2$

a) $Z = ?$ show $Z_1 \cdot Z_2$ (don't try to get number)

$$Z = \int dv_x \int dv_y \int dv_z \int dx \int dy \int dz \; e^{-E/kT}$$

$$= \left(\int_0^\infty 4\pi v^2 e^{-\frac{mv^2}{2kT}} dv\right)\left(\int_0^\infty 4\pi r^2 e^{-\frac{k}{2}r^2/kT} dr\right)$$

b) $\bar{v} = ?$

$$\bar{v} = \frac{1}{Z_1}\int_0^\infty 4\pi v^3 e^{-\frac{mv^2}{2kT}} dv$$

because r^2 part cancels out