PHYS 317 Fall 2018

# Summary for Exam 3

## Canonical Distribution:

- Derive $P(s) = \frac{\mathrm{e}^{-E(s)/(k_{\mathrm{B}}T)}}{Z}$ and similar probabilities
- Apply: $\overline{X} = \sum_s X(s)P(s)$
  - Three state system
  - Hydrogen
  - Show $\overline{E} = -\frac{\partial \ln Z}{\partial \beta}$ and fluctuation dissipation relations
  - $Z \longrightarrow \overline{E} \longrightarrow S$
  - rotations
  - paramagnet
  - derive equipartition theorem
  - Maxwell speed distribution and velocity averages and maximum
  - composite systems
- $F = -k_{\mathrm{B}}T \ln Z$
  - $Z \longrightarrow F \longrightarrow \mu, S, p$
  - ideal gas

## Ising Ferromagnet:

- derive 1 dim. $Z$ and $U$

# Formulae for Exam #3

$$k = 1.381 \cdot 10^{-23}\ \mathrm{J/K} = 8.617 \cdot 10^{-5}\ \mathrm{eV/K} \qquad h = 6.626 \cdot 10^{-34}\ \mathrm{Js} = 4.136 \cdot 10^{-15}\ \mathrm{eVs}$$

$$N_\mathrm{A} = 6.022 \cdot 10^{23} \qquad c = 2.998 \cdot 10^{8}\ \mathrm{m/s}$$

$$\Delta U = Q + W_\mathrm{on} \quad \text{where} \quad W_\mathrm{on} = -\int p\mathrm{d}V$$

$$pV = NkT \qquad pV^\gamma = \text{const., where } \gamma = (f+2)/f$$

$$U = \frac{f}{2}NkT \qquad \frac{1}{2}kT \text{ for each quadratic degree of freedom}$$

$$\Omega = \begin{pmatrix} q+N-1 \\ q \end{pmatrix} \qquad \Omega = \begin{pmatrix} N \\ n \end{pmatrix} = \frac{N!}{n!(N-n)!}$$

$$\ln N! \approx N \ln N - N \qquad \ln(1+x) \approx x$$

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

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$$C_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \qquad C_p = T\left(\frac{\partial S}{\partial T}\right)_{p,N} \qquad C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} \qquad \Delta S = \int \frac{C\mathrm{d}T}{T}$$

$$dU = TdS - pdV + \mu dN$$

$$H = U + pV \qquad F = U - TS \qquad G = U - TS + pV$$

$$S = k \ln \Omega \qquad \mathrm{d}S \geq 0 \qquad \mathrm{d}S \geq \frac{Q}{T}$$

$$G = \mu N \qquad p = \frac{NkT}{(V-bN)} - a\frac{N^2}{V^2}$$

$$\epsilon = \frac{W}{Q_\mathrm{h}} = \frac{W_\mathrm{by}}{Q_\mathrm{h}} \qquad \mathrm{COP} = \frac{Q_\mathrm{c}}{W} \qquad \frac{\mathrm{d}p}{\mathrm{d}T} = \frac{\Delta S}{\Delta V} = \frac{L}{T\Delta V}$$

$$\cosh(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} \qquad \sinh(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2} \qquad \tanh(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}} \qquad \frac{\mathrm{d}\tanh(x)}{\mathrm{d}x} = \frac{1}{\cosh^2(x)}$$

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$$\overline{X} = \sum_s X(s)\, P(s)$$

$$P(s) = \frac{\mathrm{e}^{-E(s)/(kT)}}{Z} \qquad \overline{X} = \frac{\sum_s X(s)\mathrm{e}^{-E(s)/(kT)}}{Z} \qquad Z = \sum_s \mathrm{e}^{-E(s)/(kT)}$$

$$\overline{E} = -\frac{1}{Z}\frac{\partial}{\partial \beta}(Z) \qquad F = -kT\ \ln Z$$

$$D(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \mathrm{e}^{-mv^2/(2kT)}$$

$$Z_\mathrm{idealgas} = \frac{1}{N!}\left(\frac{VZ_\mathrm{int}}{v_\mathrm{Q}}\right)^N \text{ where } v_\mathrm{Q} = \left(\frac{h}{\sqrt{2\pi mkT}}\right)^3$$

$$\int_0^\infty \mathrm{e}^{-ax^2}\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}} \qquad \int_0^\infty x^2\mathrm{e}^{-ax^2}\mathrm{d}x = \frac{\sqrt{\pi}}{4}a^{-3/2} \qquad \int_0^\infty x^4\mathrm{e}^{-ax^2}\mathrm{d}x = \frac{3}{8}\sqrt{\frac{\pi}{a^5}} \qquad \sum_{n=0}^\infty x^n = \frac{1}{1-x}$$