Advanced Classical Mechanics PHYS 331 Fall 2007

Homework Assignment #4

(due: Wednesday, September 26, 4 pm)

1. Fermat's Principle: Taylor problem (6.3)
For this problem you use that the shortest connection between two points is a straight line (which was a result obtained by using Euler-Lagrange equations). Follow the hints of Taylor (there is no need of using Euler-Lagrange equations again). (7P)

2. Set Up of Integral: Taylor problem (6.6)
Hint: for example for a) $y = y(x)$ means that we use cartesian coordinates and that we write the integral as $\int \ldots \mathrm{d}x$. So for a) the solution is $\mathrm{d}s = \sqrt{1 + y'^2}\mathrm{d}x$. (7P)

3. First Integral: Taylor problems (6.10) and (6.20) (6P)

4. Maximum Area: Taylor problem (6.22)
Hint: Follow the hints of Taylor. When you use the first integral of problem 3. you should obtain in this case $\frac{y}{\sqrt{1-y'^2}}$ =const.. It turns out that this constant is R the radius of the semicircle. Next goal will be to obtain $y(s)$. To do so solve $\frac{y}{\sqrt{1-y'^2}} = R$ for $y'(s)$. Then solve the resulting DE for $y(s)$ by using the separation of variables. (You will need $\int \frac{\mathrm{d}z}{\sqrt{1-z^2}} = \arcsin z$). You should get $y = R \sin\left(\frac{s}{R}\right)$. Then use also that $\mathrm{d}x = \sqrt{1 - y'^2}\, \mathrm{d}s$. Integrate this equation to get some expression for x. Finally put your results for x and y together to show that $(x - R)^2 + y^2 = R^2$. Make a sketch to see that this is the equation for a semicircle. (8P)

5. Molecule: Taylor problem 7.8 (7P)

6. Newton's Second Law Lab:
One of the PHYS 211 Labs is about Newton's second law. The experimental set up consists of a cart with mass m_c on a track, a string which is connected to the cart, and via a pulley connected to a hanging weight m_w (see Fig. below). Assume friction is negligible.

6a. Determine the acceleration a using Lagrangian Dynamics.

6b. Determine the acceleration a using Newton's second law. (7P)

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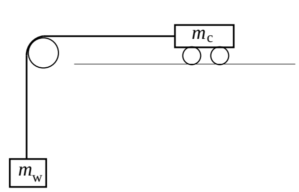

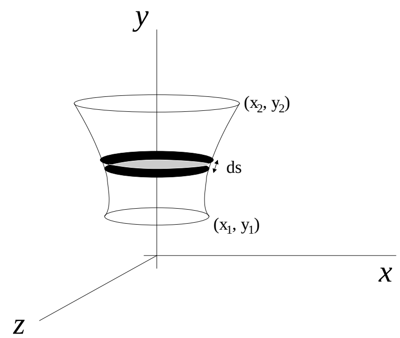

7. Soap Film: Let us consider a soap film suspended between two wire circular rings. The resulting shape of the film minimizes surface tension and thus minimizes the area. The problem for finding the shape of this surface can be restated as follows: Consider the surface generated by revolving a line connecting two fixed points (x_1, y_1) and (x_2, y_2) about the y-axis as shown in the figure above. Determine $y(x)$ which connects (x_1, y_1) and (x_2, y_2) and which minimizes the surface area generated by the revolution.
Hint: Follow the steps 7a.– 7f. (8P)

7a. Determine ds, the length of an infinitesimal segment of the path, as a function of dx and dy.

7b. Determine the area dA for the infinitesimal ring shaped area indicated in the figure above.

7c. Using your result of 7b. write the total revolving area (soap film area) as an integral with integrand $f(x, y')dx$.

7d. To minimize this area find the corresponding Euler-Lagrange equation.

7e. Solve the Euler-Lagrange equation for $y'(x)$.

7f. Now you are ready to get $y(x)$.
Hint: $\int \frac{dz}{\sqrt{z^2-1}} = \text{arcosh} z$
Note: If you rewrite this result as $x(y)$, you obtain the catenary curve, which is the curve of a freely hanging flexible cord.