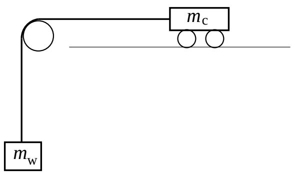
Homework Assignment #5

(due: Wednesday, October 3, 11:00 pm)

1. Molecule: Taylor problem 7.8 (HW#4 probl. 5.) (7P)

2. Newton's Second Law Lab: (HW#4 probl. 6.) (7P)



One of the PHYS 211 Labs is about Newton's second law. The experimental set up consists of a cart with mass m_c on a track, a string which is connected to the cart, and via a pulley connected to a hanging weight m_w (see Fig. above). Assume friction is negligible.

2a. Determine the acceleration a using Lagrangian Dynamics.

2b. Determine the acceleration a using Newton's second law.

3. Friction: Taylor problem (7.12) (4P)
This problem addresses the case of having a nonconservative force (friction) in addition to a conservative force. Notice that you derive an equation in cartesian coordinates which does not give you Hamilton's principle and thus we unfortunately lost the free choice of general coordinates.

4. Lagrange's Equations: Taylor problem (7.13) (10P)
Hint: For your proof you will need that for $|\vec{\epsilon}_1| \ll 1$ and $|\vec{\epsilon}_2| \ll 1$ you can approximate $U(\vec{r}_1 + \vec{\epsilon}_1, \vec{r}_2 + \vec{\epsilon}_2) = U(\vec{r}_1, \vec{r}_2) + \epsilon_1 \cdot \vec{\nabla}_1 U + \epsilon_2 \cdot \vec{\nabla}_2 U$.

5. Inclined Plane: Taylor problem (7.16) (6P)
Keep the moment of inertia as general I, i.e. do not replace I with the specific I of a uniform cylinder.

6. Pendulum with Pivot Point on Wheel: Taylor problem (7.29) (10P)
Hint: Follow Taylor's hint: Use a cartesian coordinate system with origin in point O. Express x and y of the mass as functions of ω, ϕ, t, R and l.

7. Bead on Wire: Taylor problem (7.35) (10P)
Hint: The hint of Taylor means simply that you have to be very careful in first finding $x(t)$ and $y(t)$ expressed with ω, R and ϕ (and t). Make a good sketch and be careful. You should obtain from the Euler-Lagrange equation that $\ddot{\phi} = -\omega^2 \sin\phi$. Also use that $\sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta) = \cos(\alpha - \beta)$.

8. Coin on Cone: Taylor problem (7.38) (10P)
Hints: Notice that you use now spherical coordinates (not cylindrical coordinates as in class). For finding r_0 in part b) use the same logic as on page 261 for $\theta(t)$. For part c) put $(r(t) = r_0 + \epsilon(t))$ into your equation for $\ddot{r}(t)$ from part b). This gives you an equation for $\epsilon(t)$. Approximate this equation for small $\epsilon(t)$ (you will use a Taylor series for $\frac{1}{(1+z)^3}$).