Driven Damped Pendulum

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2 \sin\phi = \gamma\omega_0^2 \cos(\omega_{\mathrm{D}} t) \tag{1}$$

Goal: $\phi(t)$, phase-space plots, Poincaré plots

We rewrite Eq. (1) as two DEs of first order for $\phi(t)$ and $\omega(t)$:

$$\dot{\phi}(t) = \omega(t) \tag{2}$$

$$\dot{\omega}(t) + 2\beta\omega(t) + \omega_0^2 \sin(\phi(t)) = \gamma\omega_0^2 \cos(\omega_{\mathrm{D}} t) \tag{3}$$

Use the same parameters as Taylor in Chapter 12:
$\omega_{\mathrm{D}} = 2\pi$, $\omega_0 = 1.5$, $\beta = \omega_0/4$, $\phi(0) = -\pi/2$, $\dot{\phi}(0) = 0$

Copy the notebook "Sept10_short.nb" from my public space into your public or private space. Save your version of the notebook frequently during this lab.

1. Plot $\phi(t)$ for times $0 \le t \le 10$ and $50 \le t \le 70$

(i) $\gamma = 1.06$ (already in notebook)
(ii) $\gamma = 1.078$
(iii) $\gamma = 1.081$
(iv) $\gamma = 1.24$

What would you expect in each case according to the bifurcation diagrams in Figs. (12.17) & (12.18)?

2. Make phase-space plots and interpret your results for

(i) $\gamma = 1.06$ for $0 \le t \le 10$ and $60 \le t \le 70$
(ii) $\gamma = 1.078$ for $60 \le t \le 70$
(iii) $\gamma = 1.081$ for $60 \le t \le 70$
(iv) $\gamma = 1.24$ for $60 \le t \le 70$

3. Make Poincaré section plots for

(i) $\gamma = 1.078$ (already done in notebook)
(iii) $\gamma = 1.081$
(iv) $\gamma = 1.24$

What is the relation between your results of 1.,2. and 3.?

Logistic Map

$$x_{n+1} = rx_n(1 - x_n) = f(x_n) \quad (4)$$

4. Plot $x(t)$ for $x_0 = 0.1$, $1 \leq t \leq 100$, and
(i) $r = 2.0$ (already done in notebook)
(ii) $r = 3.2$
Explain your results of (i) & (ii)

5. Make a bifurcation diagram for $0.5 \leq r \leq 4.0$ in steps of $\Delta r = 0.01$. You will need the mathematica commands of `For`, `Table` and `ListPlot` as they have been used in 4. You may also need the command `If`, which you can look up with the Help Menue.

Reading Assignment #9

(due: Wednesday, September 12, 8 am)
(my email: kvollmay@bucknell.edu)

Announcement:

- Wednesday, Sept. 12, we will be back in Olin 264 (as usual)
- Homework #3 will be due Friday, Sept. 14

Read: Taylor pages 503 – 514

1. Problem 6. of Homework #3 (Finish Mathematica Lab). No hand-in is required. This problem is to familiarize you with the logistic map before we do theory of it on Wednesday. Answer to this question should be "done."

2a. Define a fixpoint of a function $f(x)$.
2b. How can you determine a fixpoint graphically of any function $f(x)$?

3. Comments: What of this reading did you find most difficult and what did you find most interesting? Is there a specific topic you would like to focus on, on Wednesday in class?