Driven Damped Pendulum

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2 \sin\phi = \gamma\omega_0^2 \cos(\omega_{\rm D} t) \tag{1}$$

Goal: $\phi(t)$, phase-space plots, Poincaré plots

We rewrite Eq. (1) as two DEs of first order for $\phi(t)$ and $\omega(t)$:

$$\dot{\phi}(t) = \omega(t) \tag{2}$$
$$\dot{\omega}(t) + 2\beta\omega(t) + \omega_0^2 \sin(\phi(t)) = \gamma\omega_0^2 \cos(\omega_{\rm D} t) \tag{3}$$

Use the same parameters as Taylor in Chapter 12:
$\omega_{\rm D} = 2\pi$, $\omega_0 = 1.5\omega_{\rm D}$, $\beta = \omega_0/4$, $\phi(0) = -\pi/2$, $\dot{\phi}(0) = 0$

Copy the notebook "Sept12_short.nb" from my public space (kvollmay → public → phys331 → Sept12_short.nb) into your public or private space. Save your version of the notebook frequently during this lab.

1. Plot $\phi(t)$ for times $0 \leq t \leq 10$ and $50 \leq t \leq 70$
(i) $\gamma = 1.06$ (already in notebook)
(ii) $\gamma = 1.078$
(iii) $\gamma = 1.081$
(iv) $\gamma = 1.24$
How does this fit with our table from last class?

2. Make phase-space plots and interpret your results for
(i) $\gamma = 1.06$ for $0 \leq t \leq 10$ and $60 \leq t \leq 70$
(ii) $\gamma = 1.078$ for $60 \leq t \leq 70$
(iii) $\gamma = 1.081$ for $60 \leq t \leq 70$
(iv) $\gamma = 1.24$ for $60 \leq t \leq 70$
Hint: Use `ParametricPlot`. In case the size ratio is too narrow use `AspectRatio` → `Full`.

3. Make Poincaré section plots for
(i) $\gamma = 1.078$ (already done in notebook)
(iii) $\gamma = 1.081$
(iv) $\gamma = 1.24$
What is the relation between your results of 1.,2. and 3.?