SUMMARY

Newton's Laws

- Newton's 2^{nd} & 3^{rd} Law
- Air Resistance (Separation of variables)
- Conservation of Linear & Angular Momentum (Proofs & Appl.)
- Center of Mass

Energy & Potential

- Work $W = \int \vec{F} \cdot d\vec{r}$
- many particle systems $\underline{F}_i = -\underline{\nabla}_i U$

Harmonic Oscillator

- SHO, DHO, DDHO (solution with e^{rt}, sketch, resonance principle)

Non-Linear Dynamics

- DDP
- $\phi(t) \longleftrightarrow \dot{\phi}(\phi)$ or $p(q) \longleftrightarrow$ Poincaré Plot $\longleftrightarrow$ Bifurc.
- Logistic Map: Fix point, Stability, 2-period fix pt.

Variational Principle

① $S = \int \ldots$

② $S = \int f(x, x', y, y', u)\, du$

③ EL Eq. $\frac{\partial f}{\partial x} = \frac{d}{du} \frac{\partial f}{\partial x'}$ $\quad \frac{\partial f}{\partial y} = \frac{d}{du} \frac{\partial f}{\partial y'}$

④ Solve (& interpret)

Lagrange Method

① $\mathcal{L} = \sum_i T_i - U$ ⓵a cartes. c. ⓵b general. coord.

② ELE $\frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i}$

③ Solve

Central Force, 2 Body Problem

- $\mathcal{L} = \mathcal{L}_{cm} + \mathcal{L}_{rel}$
- $U_{eff} \rightarrow$ orbits ; $r(\phi) \rightarrow$ ellipse etc.

Non-Inertial Frames

- translational $\underline{A}$ $m\ddot{\underline{r}} = \underline{F} - m\underline{A}$
- $m\ddot{\underline{r}} = \underline{F} + \underline{F}_{cor} + \underline{F}_{cf} + \underline{F}_{azim}$

proof & application

Rotation of Rigid Bodies

- $\underline{F}_{ext} = M\ddot{\underline{R}}_{cm}$
- $\underline{L} = \underline{\underline{I}}\,\underline{\omega}$ (derive & apply)

principal axes & eigenvalues:

① determine $\underline{\underline{I}}$

② $|\underline{\underline{I}} - \lambda \underline{\underline{1}}| = 0 \rightarrow \lambda_1, \ldots$

③ $(\underline{\underline{I}} - \lambda \underline{\underline{1}})\,\underline{\omega} = 0 \rightarrow \underline{\omega}_1, \ldots$

Coupled Oscillators

$$\underline{\underline{M}}\,\ddot{q} = -\underline{\underline{K}}\,q$$

① DEs (Newton, $\mathcal{L}$, approx.)

② $\underline{\underline{M}}$, $\underline{\underline{K}}$

③ ansatz: $q = \mathrm{Re}(\underline{a}\, e^{i\omega t})$

$$(\underline{\underline{K}} - \omega^2 \underline{\underline{M}})\,\underline{a} = 0$$

④ $|\underline{\underline{K}} - \omega^2 \underline{\underline{M}}| = 0 \quad \rightarrow \omega_1, \ldots$

⑤ $(\underline{\underline{K}} - \omega^2 \underline{\underline{M}})\,\underline{a} = 0 \quad \rightarrow \underline{a}_1, \ldots$

⑥ describe Motion

Hamiltonian Mechanics

① T, U (1a) cartes.   (1b) general. coord.

$\rightarrow \mathcal{L} = T - U$

② $p_i = \dfrac{\partial \mathcal{L}}{\partial \dot{q}_i} \quad \rightarrow \dot{q}_i(q, p, t)$

③ $\mathcal{H} = \sum_i p_i \dot{q}_i - \mathcal{L}$ (if natural $\mathcal{H} = T + U$)

④ $\dot{q}_i = \dfrac{\partial \mathcal{H}}{\partial p_i} \qquad \dot{p}_i = -\dfrac{\partial \mathcal{H}}{\partial q_i}$

applications, canon. transf., Liouville Thm.