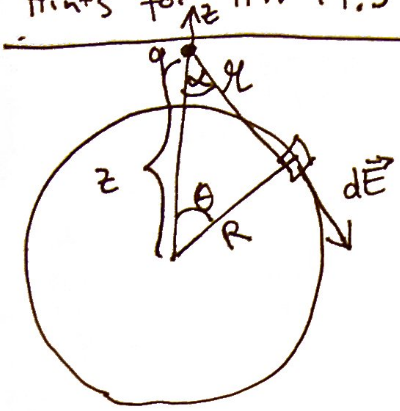
Hints for HW 14.3 Griffiths 3.4



by symmetry

$$\vec{E}_{ave} = E_{ave,z}\,\hat{z} = -E_{ave}\,\hat{z}$$

so $$E_{ave,z} = -\frac{1}{4\pi R^2}\underbrace{\oint}_{\text{sphere surface}} dE_z\, da = -\frac{1}{4\pi R^2}\oint \overbrace{dE}^{\left(\frac{q}{4\pi\varepsilon_0}\frac{1}{𝓇^2}\right)} \cos\alpha\, \underbrace{da}_{R^2\sin\theta\, d\theta\, d\phi}$$

$$\int_0^{2\pi} d\phi = 2\pi$$

So we need to express all variables in terms of θ and constants R and z:

$$𝓇^2 = R^2 + z^2 - 2Rz\cos\theta$$

for $\cos\alpha$ we use $R^2 = z^2 + 𝓇^2 - 2𝓇 z\cos\alpha \rightarrow$ solve for $\cos\alpha$

$\downarrow$ put into $E_{ave,z}$ above

$\downarrow$ ... & simplify

$$E_{ave,z} = -\frac{q}{16\pi^2 R^2\varepsilon_0}\int_0^{\pi} \frac{z - R\cos\theta}{(R^2+z^2-2Rz\cos\theta)^{3/2}} R^2\sin\theta\, d\theta\, d\phi$$

$\downarrow$ substitution $\tilde{u} = \cos\theta$ $\quad \frac{d\tilde{u}}{d\theta} = -\sin\theta$

$$E_{ave,z} = -\ldots\int_{+1}^{-1} \frac{(z - R\tilde{u})}{(R^2+z^2-2Rz\tilde{u})^{3/2}} R^2 \cancel{\sin\theta}\frac{d\tilde{u}}{(-\cancel{\sin\theta})}$$

$\downarrow$ use integration by parts

use $u = (z - R\tilde{u})$ $\quad v' = (R^2+z^2-2Rz\tilde{u})^{-3/2}$

$\downarrow$ ...

then distinguish $z > R$ and $z < R$ for taking $\sqrt{\ldots}$ to get positive value