Oct. 28

Electrostatic: $\vec{E} = -\vec{\nabla} V$

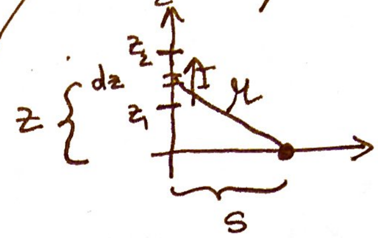
Magnetostatic: $\vec{B} = \vec{\nabla} \times \vec{A}$

Read §6.1 (pages 266-273)

& For next class: Bound. Cond. & Multipole Exp. for Summary

HW#29 5.23, 5.24, 5.26

Hint:



$\vec{A} \to \vec{B} \to \vec{J}$

in (a) and (b)

$\vec{A} = A(s)\hat{z}$

(a) we know $\vec{B}$, so

$\vec{\nabla} \times \vec{A} = \vec{B}$

integrate to get $\vec{A}$

(b) first use Ampère's law to get $\vec{B}$

then use approach as in (a)

get constants from integration via $\vec{A}_{\text{inside}}(s=R) = \vec{A}_{\text{outside}}(s=R)$

Motivation for Potentials:

- easier equations (more obvious for V which is scalar)
- for boundary conditions (used for method of images & separation of variables)
- (Ch 10) For special relativity and time dependent $\vec{E}$ & $\vec{B}$:

$\begin{pmatrix} V \\ \vec{A} \end{pmatrix}$ are 4-vector

(Eqs. (10.25), (10.26) page 445 and (10.34) page 449)

$$t_r = t - \frac{\mathscr{r}}{c}$$

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,', t_r)}{\mathscr{r}} d\tau'$$

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,', t_r)}{\mathscr{r}} d\tau'$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

depending on how far we get on HW29 or HW30:

5.35, 5.36, 5.41

Maxwell Equations:

$$\boxed{\square^2 A^\mu = -\mu_0 J^\mu}$$

(12.137) page 570

transforms correctly, much more elegant, captures E&M in one-liner!

Fill in Bound. Cond. & Multip. Expansion ↓

Electrostatics

ρ V $\vec{E}$

$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}}\, d\tau$

$\vec{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}^2}\, d\tau\, \hat{\mathscr{r}}$

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ $\quad \vec{\nabla}\times\vec{E} = 0$

Gauss's Law $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$

$\vec{E} = -\vec{\nabla} V$

$V = -\int \vec{E}\cdot d\vec{l}$

Magnetostatics

$\vec{J}$ $\vec{A}$ $\vec{B}$

$K\,da$

$I\,d\vec{l}$

only if current finite

$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{\mathscr{r}}\, d\tau}$

$\boxed{\vec{\nabla}^2\vec{A} = -\mu_0 \vec{J}}$

$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau$

Ampere's Law $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

$\vec{\nabla}\cdot\vec{B} = 0$

$\boxed{\vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{\nabla}\cdot\vec{A} = 0}$

§5.4