Force $\vec{F}_{elec} = Q\vec{E}$

Electrostatics

ρ, $\vec{E}$, V

$\rho \to \vec{E}$: $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}^2}\, d\tau\, \hat{\mathscr{r}}$

$\vec{E} \to \rho$: $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$, $\vec{\nabla}\times\vec{E} = 0$

Gauss's law $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$

$\rho \to V$: $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}}\, d\tau$

$V \to \rho$: $\vec{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$

$V \to \vec{E}$: $\vec{E} = -\vec{\nabla}V$

$\vec{E} \to V$: $V = -\int \vec{E}\cdot d\vec{l}$

(p.88 & 89) **Boundary Conditions**

$$\left.\begin{aligned} E^{\perp}_{above} - E^{\perp}_{below} &= \frac{\sigma}{\epsilon_0} \\ \vec{E}^{\parallel}_{above} &= \vec{E}^{\parallel}_{below} \end{aligned}\right\} \vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$V_{above} = V_{below}$$

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\epsilon_0}$$

(p.185) $D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$

$\vec{D}^{\parallel}_{above} - \vec{D}^{\parallel}_{below} = \vec{P}^{\parallel}_{above} - \vec{P}^{\parallel}_{below}$

Multipole Expansion (p.153 ff)

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int (r')^n P_n(\cos\alpha)\rho(\vec{r}')\,d\tau'$$

$$V_{mon}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}\cdot\underbrace{\int \vec{r}'\rho(\vec{r}')\,d\tau'}_{\vec{p}\,=\,\text{electric dipole}}$$

$$\vec{E}_{dip} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$\hat{z}$ $\vec{p}$: $\vec{E}_{dip} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

for point charges $\vec{p} = \sum_{i=1}^{n} q_i\vec{r}_i'$

$\vec{N} = \vec{p}\times\vec{E}$

$\vec{F}_{dip} = (\vec{p}\cdot\vec{\nabla})\vec{E}$

Magnetostatics

$\vec{J}$, $\vec{B}$, $\vec{A}$

$\vec{J} \to \vec{B}$: $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau$

$\vec{B} \to \vec{J}$: $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ Ampere's Law $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{J} \to \vec{A}$: $\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{\mathscr{r}}\,d\tau}$ ($\vec{K}\,da$, $I\,d\vec{l}$) only if current finite

$\vec{A} \to \vec{J}$: $\boxed{\nabla^2\vec{A} = -\mu_0\vec{J}}$

$\vec{A} \to \vec{B}$: $\boxed{\vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{\nabla}\cdot\vec{A} = 0}$

Forces $\vec{F}_{mag} = Q\vec{v}\times\vec{B}$

$\vec{F}_{mag} = \int I\,d\vec{l}\times\vec{B}$

OR

$\int \vec{K}\times\vec{B}\,da = \vec{F}_{mag}$

OR

$\int \vec{J}\times\vec{B}\,d\tau = \vec{F}_{mag}$

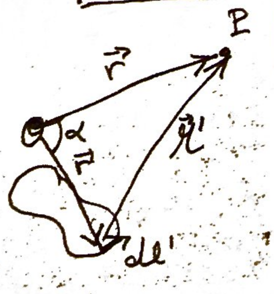
§5.4

Boundary Conditions (p.250)

$$\left.\begin{aligned} B^{\perp}_{above} &= B^{\perp}_{below} \\ B^{\parallel}_{above} - B^{\parallel}_{below} &= \mu_0 K \end{aligned}\right\} \vec{B}_{above} - \vec{B}_{below} = \mu_0(\vec{K}\times\hat{n})$$

$$\vec{A}_{above} = \vec{A}_{below}$$

$$\frac{\partial\vec{A}_{above}}{\partial n} - \frac{\partial\vec{A}_{below}}{\partial n} = -\mu_0\vec{K}$$



Multipole Expansion (p.252 ff)

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\oint (r')^n P_n(\cos\alpha)\,d\vec{l}'$$

$\hat{z}$ $\vec{m}$: $\vec{B}_{dip}(\vec{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right] \qquad \vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$\vec{m} = I\int d\vec{a}$ magnetic moment

$\vec{N} = \vec{m}\times\vec{B}$ $\qquad \vec{F}_{dip} = \vec{\nabla}(\vec{m}\cdot\vec{B})$