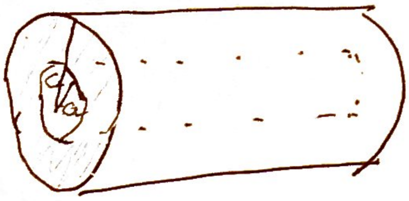
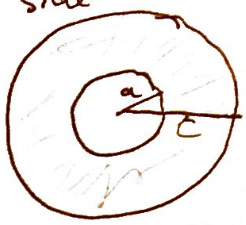
Example ← similar to probl. 4.15

A long pipe of dielectric material with frozen in $\vec{P}(s) = k s^4 \hat{s}$ & no free charge. here for $a<r<c$

$\vec{E}$ via two routes part (a) & (b)
↑ Gauss's Law for $\vec{E}$ ↑ for $\vec{D}$



side view

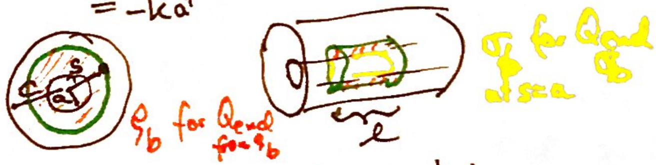


a) $\rho_b = ?$ $\sigma_b = ?$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{s}\frac{\partial}{\partial s}\left(s\,k s^4\right) = -\frac{1}{s} k\,5 s^4 = -5 k s^3$$

↑ $P_\phi = P_z = 0$

$$\sigma_{b\,at\,a} = \vec{P}\cdot\hat{n} = P\hat{s}\cdot(-\hat{s}) = -P = -ks^4\Big]_{at\,a} = -ka^4 \quad (\&\ \sigma_{b\,at\,c} = P)$$



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\int(\rho_b + \underbrace{\rho_f}_{=0})\,d\tau' + \frac{1}{\varepsilon_0}\int_{\text{wrap at a}} \underbrace{\sigma_{b\,at\,a}}_{\text{const. at s}}\,da'$$

$$= \frac{1}{\varepsilon_0}\int_a^s -5ks'^3\,2\pi s'\ell\,ds' + \frac{1}{\varepsilon_0}\,(-P)\,2\pi s\ell$$

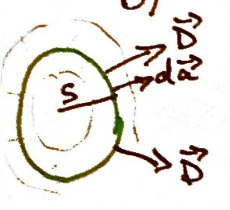
$\oint \ldots = \int_{wrap} \ldots + \underbrace{\int_{caps} \ldots}_{=0 \text{ because } \vec{E}\perp d\vec{a}}$

$$E\,2\pi s\ell = -\frac{1}{\varepsilon_0}\,5k\,2\pi\ell\left[\frac{s^5}{5}\right]_a^s + \frac{1}{\varepsilon_0}\left(-ka^4\right)2\pi a\ell$$

$$E\,s = -\frac{1}{\varepsilon_0}k\left(s^5 - a^5\right) - \frac{1}{\varepsilon_0}k a^5$$

$$\hookrightarrow \vec{E} = -\frac{1}{\varepsilon_0}k s^5 \frac{1}{s}\hat{s} = \boxed{-\frac{1}{\varepsilon}k s^4 \hat{s}}$$

b) $\oint \vec{D}\cdot d\vec{a} = \frac{Q_{f\,encl}}{\varepsilon_0}$



D const. on Gauss. Surf.

$D\,2\pi s\ell = 0$

$\rightarrow \vec{D} = \vec{0}$ ↗ $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$$\vec{E} = -\frac{1}{\varepsilon_0}\vec{P} = \boxed{-\frac{1}{\varepsilon_0}k s^4 \hat{s}}$$