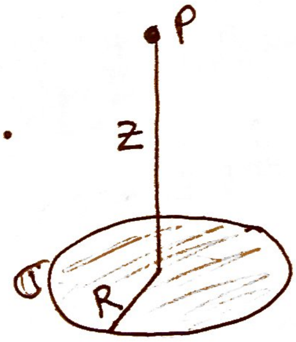
Jeopardy

I Vector Analysis	II Electrostatics
I1. 1C	2C
I2. 1C	2C

I1. Determine $\int_{-\infty}^{6} (8+x^2)\,\delta(6-3x)\,dx$

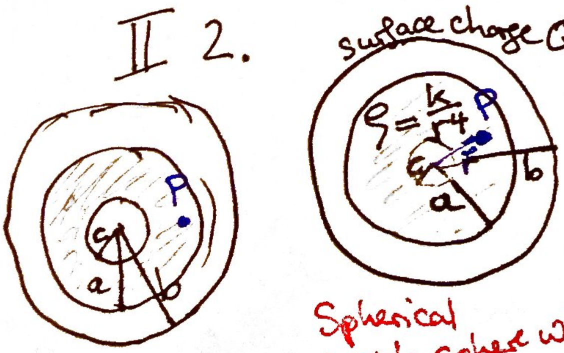
I2. Determine $\vec{\nabla}^2 (9x^2y + 4z^3)$

II 1.

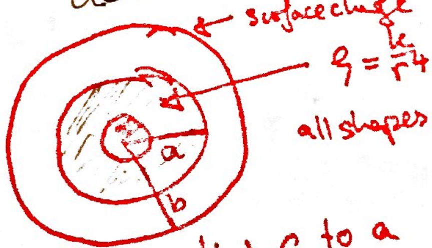


Write an integral expression for E_z in point P.

II 2.



Use Gauss's Law to determine $\vec{E}$ in point P at $c<r<b$



all shapes spherical

Spherical
inside sphere with hole from radius c to a
outside surface charge at radius b

I1. Determine $\displaystyle\int_{-\infty}^{6}(8+x^2)\,\underbrace{\delta(6-3x)}_{\frac{1}{3}\delta(2-x)}\,dx$

$$\int_{-\infty}^{6}(8+x^2)\,\frac{1}{3}\,\delta(\underbrace{2-x}_{x=2})\,dx \underset{\text{in integr. range}}{=} \frac{1}{3}(8+2^2) = \boxed{\frac{12}{4}} = \boxed{3}$$

$\vec{\nabla}\cdot(\vec{\nabla}T) = \vec{\nabla}^2 T$

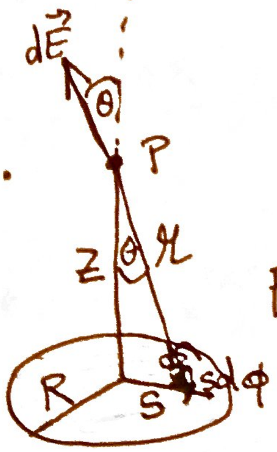
I2. $\vec{\nabla}^2(9x^2y+4z^3)$

$$\frac{\partial^2}{\partial x^2}(9x^2y+4z^3)+\frac{\partial^2}{\partial y^2}(9x^2y+4z^3)+\frac{\partial^2}{\partial z^2}(9x^2y+4z^3)$$

$$= 18y \quad + \quad 0 \quad + \quad 24z$$

Coulombs law $\vec{E} = \int \frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}$

II1.



$$dE_z = dE\,\underbrace{\cos\theta}_{z/\mathscr{r}}$$

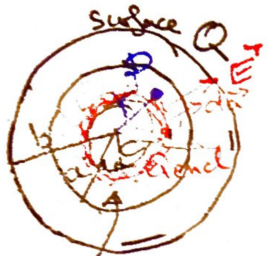
$$E_z = \int_0^R\int_0^{2\pi}\frac{\sigma\overbrace{(ds\,s\,d\phi)}^{da}}{\mathscr{r}^2}\cos\theta$$

$$E_z = \int_0^R ds\int_0^{2\pi} s\,d\phi\,\frac{\sigma}{\mathscr{r}^2}\cos\theta$$

$$= \int_0^R ds\;2\pi\sigma\,\frac{s\,z}{(z^2+s^2)^{3/2}}$$

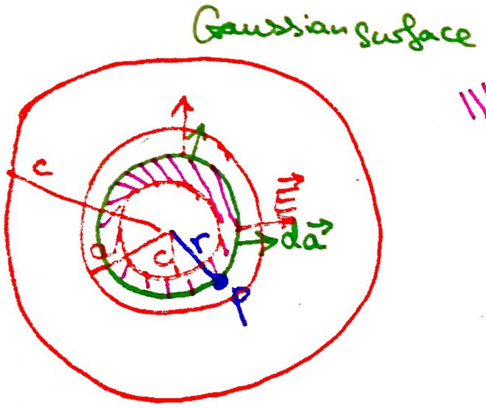
II 2.

Gauss's Law



$$\rho = \frac{k}{r^4}$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$$



IIII = Q_{encl}

$$E(4\pi r^2) = \frac{\int_c^r (4\pi r'^2) \frac{k}{(r')^4} dr'}{\varepsilon_0}$$

$$= \frac{4\pi k}{\varepsilon_0} \int_c^r \frac{dr'}{(r')^2}$$

$$= \frac{4\pi k}{\varepsilon_0} \left[-\frac{1}{r'} \right]_c^r$$

$$E\, 4\pi r^2 = \frac{4\pi k}{\varepsilon_0} \left[\frac{1}{c} - \frac{1}{r} \right]$$

$$\boxed{\vec{E} = \frac{k}{\varepsilon_0 r^2} \left(\frac{1}{c} - \frac{1}{r} \right) \hat{r}}$$

Summary Ch1 & Ch2

Write your "Equation Sheet"

Vector Analysis:

$\vec{\nabla} f$, $\vec{\nabla}\cdot\vec{v}$, $\vec{\nabla}\times\vec{v}$, $\vec{\nabla}^2\vec{v}$ determine (cartesian & curvilinear)

be able to do line, surface, and volume integral

Fundamental Theorems:

$$\int_{\vec{a}}^{\vec{b}} (\vec{\nabla} T)\cdot d\vec{l} = T(\vec{b}) - T(\vec{a})$$

$$\int_{\mathcal{V}} (\vec{\nabla}\cdot\vec{v})\, d\tau = \oint_{S} \vec{v}\cdot d\vec{a}$$ diverg. thm, Gauss's thm, Green's thm

$$\int_{S} (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \oint_{P} \vec{v}\cdot d\vec{l}$$ Stoke's thm

$$\boxed{\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}}$$

$\delta(x)$, $\delta^3(\vec{r})$

$\vec{\nabla}\times\vec{F} = 0 \iff \dots \iff \vec{F} = -\vec{\nabla} V$

$\vec{\nabla}\cdot\vec{F} = 0 \iff \dots \iff \vec{F} = \vec{\nabla}\times\vec{a}$

Electrostatics

Coulomb's Law for points & continuous charges $\vec{F}$, $\vec{E}$

flux, Gauss's law applications, $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

$$\vec{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$$

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{l}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{0}\frac{dq}{\mathscr{r}}$$ and then $\vec{E} = -\vec{\nabla} V$

$$W = \frac{1}{2}\int_{\mathcal{V}} \rho(\vec{r})\, V(\vec{r}) = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau$$