SUMMARY FOR EXAM 2

Chs 3 & 4; HW14-24

Overall Summary: determine $V(\vec{r})$, $\vec{E}(\vec{r})$, $\vec{D}(\vec{r})$, $\sigma_b(\vec{r})$, $\rho_b(\vec{r})$, $\sigma(\vec{r})$

① Uniqueness Theorems: HW14
- ∘ for proof: $V(\vec{r}) \underset{\text{we showed}}{=} \frac{1}{4\pi R^2} \oint V\, da$ } not on exam 2
- ∘ proof theorems
- • apply (② & ③)

② Method of Images: HWs 15, 16, 20
- • find images, show bound. cond. satisfied
- • $\vec{F}_{\text{on } q}$, $\sigma = -\varepsilon_0 \frac{\partial V}{\partial n}$

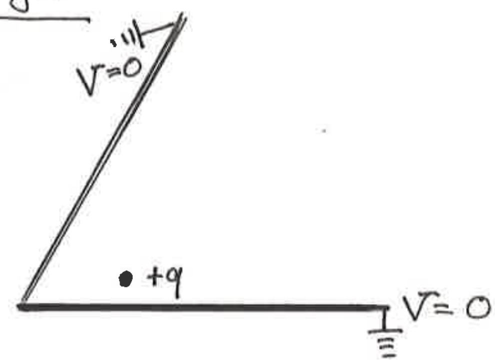
③ Separation of Variables HW 17, 18, 24, 22
- • $\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$ mostly $\vec{\nabla}^2 V = 0$
- • do separation of variables → diff. equs.
- • find general solution
- • name boundary conditions
- • find coefficients (incl. Fourier's trick)

④ Multipole Expansion: HW19
- ∘ derive } not on exam
- • determine V_{mon}, V_{dip} → $\vec{E}_{dip}$, $\vec{p}$, $\vec{F}$

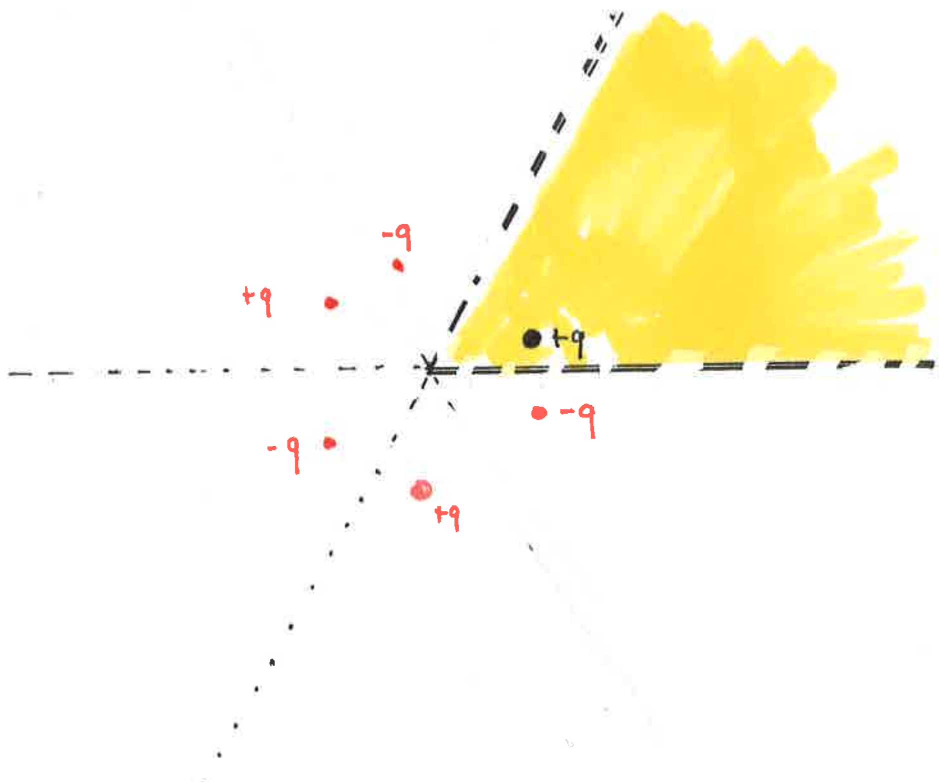
⑤ Dipole & Polarization: HW20, HW21, 22
- • $\vec{p}$, $\vec{N}$, $\vec{F}$
- • $\vec{P}$, σ_b, ρ_b → $\vec{E}$

⑥ Dielectrics HW 20 – 24
- • $\vec{D}$, Gauss's Law, $\vec{P}$, $\vec{E}$, $\varepsilon_0, \varepsilon_r, \chi_e$
- • boundary conditions for $\vec{D}^{\parallel}_{\text{above}}$ etc., $D^{\perp}_{\text{above}}$ etc., $\vec{E}^{\parallel}_{\text{above}}$, $\vec{E}^{\perp}_{\text{above}}$ etc. HW 22
- • boundary conditions for V_{above}, $\frac{\partial V_{\text{above}}}{\partial n}$ etc. HW24
- • W and $C = \frac{Q}{V}$ HW23, HW24

Method of Images

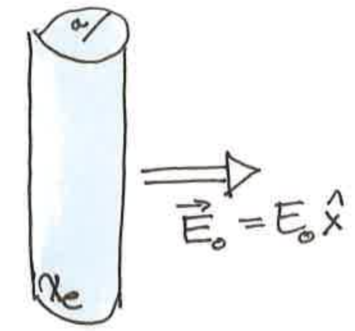


Sketch the image charge(s)

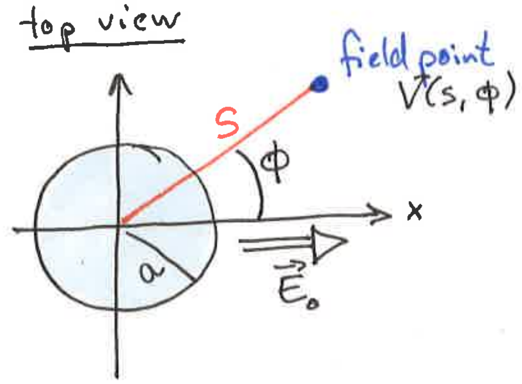


Note : → $V(\vec{r})$, $\vec{E}(\vec{r})$, $\vec{F}_{on\,q}$, $\sigma_{induced}$

Separation of Variables



long cylinder
dielectric



Derive Diff. Equs. using separation of variables

(HW#18.2) $$V(s, \phi, z) = V(s, \phi) = S(s)\, \Phi(\phi)$$

$$\vec{\nabla}^2 V = 0$$

$$\vec{\nabla}^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial S}{\partial s}\right)\Phi(\phi) + \frac{1}{s^2}\left(\frac{\partial^2}{\partial \phi^2}\Phi\right) S(s) = 0$$

$$\frac{1}{S}\, s \frac{\partial}{\partial s}\left(s \frac{\partial S}{\partial s}\right) = -\frac{1}{\Phi}\left(\frac{\partial^2}{\partial \phi^2}\Phi\right) = k^2$$

Separation of Variables (continued)

boundary conditions

(HW24.1)

(i) $V_{in} = V_{out}$

(ii) $\varepsilon_{in} \frac{\partial V_{in}}{\partial n} = \varepsilon_{out} \frac{\partial V_{out}}{\partial n}$

(iii) $V_{out}(s \to \infty) = -E_0 s \cos\theta$

$\ldots \to$

(i) $$\sum_1 a^k \left(a_k \cos(k\phi) + b_k \sin(k\phi)\right) = -E_0\, a \cos\phi + \sum_1 a^{-k}\left(c_k \cos(k\phi) + d_k \sin(k\phi)\right)$$

(ii) $$\varepsilon_r \sum_1 k a^{k-1}\left(a_k \cos(k\phi) + b_k \sin(k\phi)\right) = -E_0 \cos\phi - \sum k a^{-k-1}\left(c_k \cos(k\phi) + d_k \sin(k\phi)\right)$$

Determine b_k, d_k

Treat $\cos(k\phi)$, $\sin(k\phi)$ like $\hat{x}, \hat{y}, \hat{z}$
complete set of orthogonal solutions

$$\vec{A} = \vec{B}$$

$$A_x \hat{x} + A_y \hat{y} + A_z \hat{z} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

i) $a^k b_k = a^{-k} d_k \to d_k = a^{2k} b_k$

ii) $\varepsilon_r k a^{k-1} b_k = -k a^{-k-1} d_k \to d_k = -\varepsilon_r a^{2k} b_k = -\varepsilon_r d_k$ ↯

$\to b_k = d_k = 0$