Summary for Exam 3

HW25 – 32

$$\vec{F}_{mag} = Q\,\vec{v} \times \vec{B} \qquad \vec{F}_{mag} = \int I\, d\vec{l} \times \vec{B}$$

$$\vec{F}_{elec} = Q\,\vec{E}$$

$\vec{K}$, $\vec{J}$

Biot-Savart $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int I \frac{d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

Ampère's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \leftrightarrow \quad \int \vec{B} \cdot d\vec{a} = 0$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l}}{\mathscr{r}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{m} = I \int d\vec{a}$$

$$\vec{N} = \vec{m} \times \vec{B}$$

$$\vec{F}_{dip} = \vec{\nabla} \cdot (\vec{m} \cdot \vec{B})$$

$$\vec{M} = \frac{\vec{m}}{volume}$$

$\vec{A}(\vec{r}) = \ldots$

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{n} \Big|_{at\ surface}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

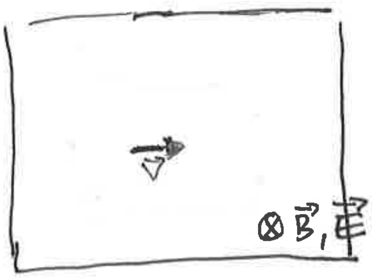
$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

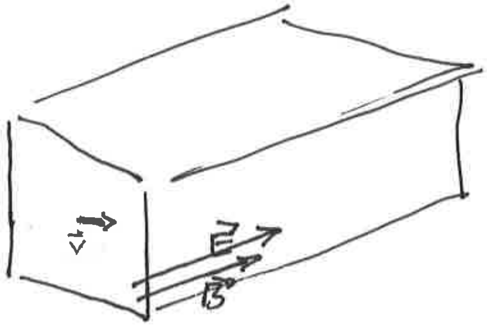
$$\oint \vec{H} \cdot d\vec{l} = I_{f,\,encl}$$

(Separation of variables not Exam 3 but Final)

Question 1

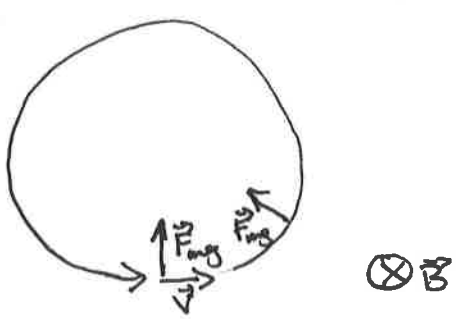
A positive charge $+q$ has velocity $\vec{v}$ at the instant shown. It is in a uniform $\vec{B}$ field and also a uniform $\vec{E}$ field. $\vec{B}$ and $\vec{E}$ are in the same direction as sketched.



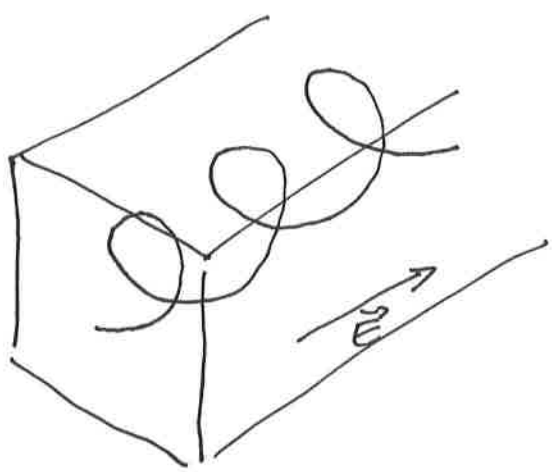


Sketch the resulting path of the $+q$ charge.

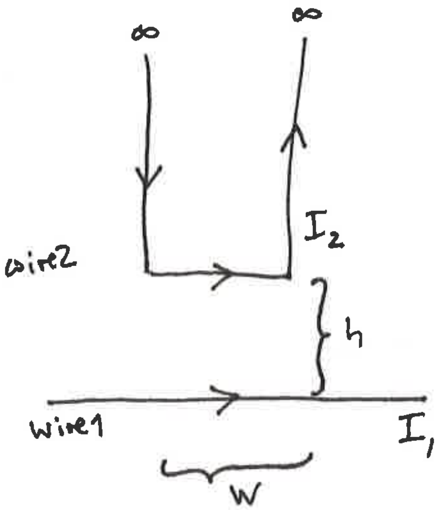
Answer to Question 1



$$\vec{F}_{mag} = +q\,\vec{v} \times \vec{B}$$



$$\vec{F}_{elec} = q\vec{E}$$

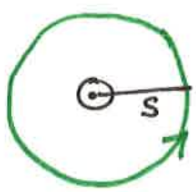


Question 2

What is the force on wire 2 (with current I_2) due to wire 1 (with current I_1)?

Answer to Question 2

$\vec{B}$ due to wire 1 :



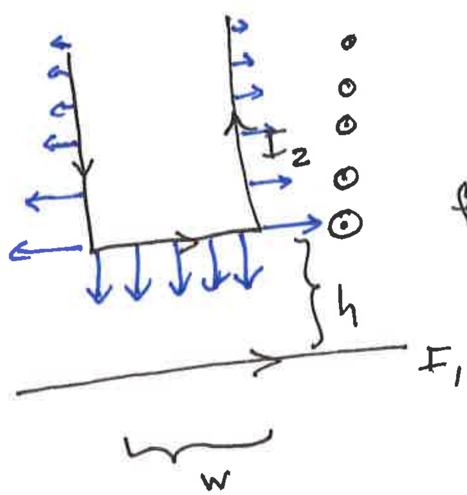
Ampère's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B\, 2\pi s = \mu_0 I_1$$

$$B = \frac{\mu_0 I_1}{2\pi s}$$

$\vec{F}_{\text{on wire 2}}$ due to $\vec{B}$:



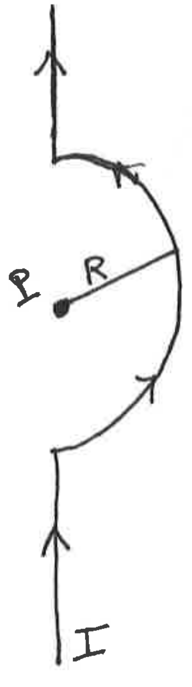
$$\vec{F}_{mag} = \int I\, d\vec{l} \times \vec{B}$$

forces due to the left & right sides cancel

the bottom wire gives

$$I_2 w B = \boxed{I_2 w \frac{\mu_0 I_1}{2\pi h}}$$

$\boxed{\text{downwards}}$



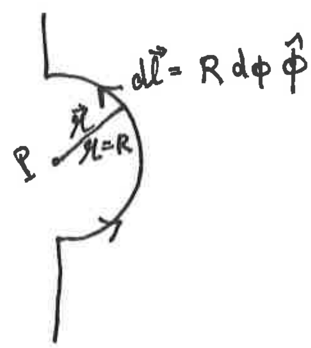
Question 3

Use Biot-Savart to determine $\vec{B}$ in the field point P

Biot-Savart:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0}{4\pi} I \int_0^{\pi} \frac{R\, d\phi}{R^2} \underbrace{\hat{\phi} \times \hat{\mathscr{r}}}_{\text{out of page, lets call this } \hat{z}}$$

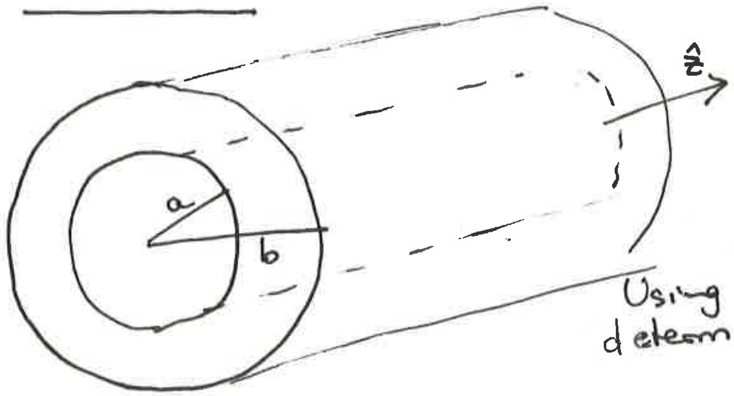
$$\vec{B}_{\text{upper}} = 0 = \vec{B}_{\text{lower}}$$



$$\vec{B}(\vec{r} = \vec{0}) = \frac{\mu_0 I}{4\pi R}\pi\, \hat{z}$$

$$= \boxed{\frac{\mu_0 I}{4R}\, \hat{z}}$$

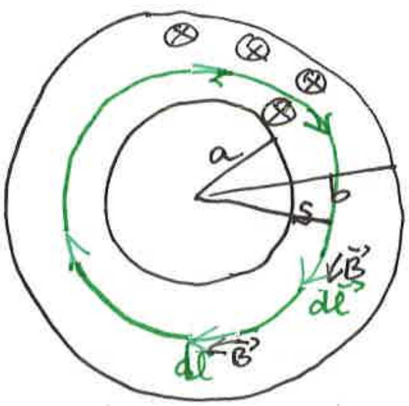
Question 4



long cylinder with

$$\vec{J} = \begin{cases} \vec{0} & \text{for } s<a \text{ \& } s>b \\ k\frac{1}{s^2}\hat{z} & \text{for } a<s<b \end{cases}$$

Using Ampère's law determine $|\vec{B}|$ for $a<s<b$



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint B\, dl \cos\theta = \mu_0 \int J\, da_\perp$$

$$B\, 2\pi s = \mu_0 \int_a^s k \frac{1}{s'^2}\, 2\pi s'\, ds'$$

$$= \mu_0 k\, 2\pi \int_a^s \frac{1}{s'}\, ds'$$

$$B\, 2\pi s = \mu_0 k\, 2\pi \ln\left(\frac{s}{a}\right)$$

$$|\vec{B}| = \boxed{\frac{\mu_0 k}{s} \ln\left(\frac{s}{a}\right)}$$

Question 5

For the vector $\vec{v} = \frac{kz}{s}\hat{\phi}$ (in cylindrical coord.)

determine $\vec{\nabla} \times \vec{v}$

$$\vec{v} = \frac{kz}{s}\hat{\phi}$$

$$\vec{\nabla}\times\vec{v} \underset{\text{cyl. coord.}}{=} \left[\frac{1}{s}\underbrace{\frac{\partial v_z}{\partial \phi}}_{=0} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\underbrace{\frac{\partial v_s}{\partial z}}_{=0} - \underbrace{\frac{\partial v_z}{\partial s}}_{=0}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \underbrace{\frac{\partial v_s}{\partial \phi}}_{=0}\right]\hat{z}$$

$$= -\frac{\partial}{\partial z}\left(\frac{kz}{s}\right)\hat{s} + \frac{1}{s}\underbrace{\frac{\partial}{\partial s}\left(s\,\frac{kz}{s}\right)}_{=0}\hat{z}$$

$$= -\frac{k}{s}\hat{s} + \cancel{\frac{1}{s}\frac{kz}{s}\hat{z}}$$

$$\vec{\nabla}\times\vec{v} = \boxed{-\frac{k}{s}\hat{s} + \cancel{\frac{kz}{s^2}\hat{z}}}$$

Question 5 (Different Question)

For the vector $\vec{V} = \frac{kz}{s^2}\hat{\phi}$ (in cylindr. coord.)

determine $\vec{\nabla} \times \vec{V}$.

Answer: $\vec{V} = \underbrace{\frac{kz}{s^2}}_{=V_\phi}\hat{\phi}$

$$\vec{\nabla} \times \vec{V} \underset{\text{cyl. coord.}}{=} \left[\frac{1}{s}\underbrace{\frac{\partial V_z}{\partial \phi}}_{=0} - \frac{\partial V_\phi}{\partial z}\right]\hat{s} + \left[\underbrace{\frac{\partial V_s}{\partial z}}_{=0} - \underbrace{\frac{\partial V_z}{\partial s}}_{=0}\right]\hat{\phi}$$

$$+ \frac{1}{s}\left[\frac{\partial}{\partial s}(s V_\phi) - \underbrace{\frac{\partial V_s}{\partial \phi}}_{=0}\right]\hat{z}$$

$$= -\frac{\partial V_\phi}{\partial z}\hat{s} + \frac{1}{s}\frac{\partial}{\partial s}(s V_\phi)\hat{z}$$

$$= -\frac{\partial}{\partial z}\left(\frac{kz}{s^2}\right)\hat{s} + \frac{1}{s}\frac{\partial}{\partial s}\underbrace{\left(s\frac{kz}{s^2}\right)}_{kz/s}\hat{z}$$

$$= -\frac{k}{s^2}\hat{s} + \frac{1}{s}kz\left(-\frac{1}{s^2}\right)\hat{z}$$

$$= -\frac{k}{s^2}\hat{s} - \frac{kz}{s^3}\hat{z}$$