Homework Assignment #33

(due Nov. 9, 2020, 11pm, via gradescope)

1. Griffiths 6.15

   Hints: Follow the hints provided in the problem statement, including to use Eq. (3.65). The hint to use Eq. (6.24) corresponds to

$$\begin{aligned} H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} &= -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}) \\ H^{\perp}_{\text{outside}} - H^{\perp}_{\text{inside}} &= (M^{\perp}_{\text{inside}} - M^{\perp}_{\text{outside}}) \\ H^{\perp}_{\text{outside}} - H^{\perp}_{\text{inside}} &= M^{\perp}_{\text{inside}} \\ -\frac{\partial W_{\text{outside}}}{\partial r}\bigg|_{r=R} + \frac{\partial W_{\text{inside}}}{\partial r}\bigg|_{r=R} &= M^{\perp}_{\text{inside}} = M\,\hat{\mathbf{z}}\cdot\,\hat{\mathbf{r}} = M\cos(\theta) \end{aligned}$$

   where the left side of the last line followed from $\mathbf{H} = -\vec{\nabla}W$ in spherical coordinates.

   To get a second boundary condition we use that

$$W(\mathbf{b}) - W(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} \vec{\nabla}W \cdot \mathrm{d}\vec{l} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{H}\cdot\mathrm{d}\vec{l}$$

   from which follows for $|\mathbf{a}-\mathbf{b}| \to 0$ that $W(\mathbf{b}) - W(\mathbf{a}) \to 0$, therefore

$$W_{\text{inside}}(r,\theta) = W_{\text{outside}}(r,\theta)$$

   All this tells you that you may use the boundary conditions

$$\begin{aligned} &(i) \quad W_{\text{inside}}(r,\theta) = W_{\text{outside}}(r,\theta) \\ &(ii) \quad -\frac{\partial W_{\text{outside}}}{\partial r}\bigg|_{r=R} + \frac{\partial W_{\text{inside}}}{\partial r}\bigg|_{r=R} = M^{\perp}_{\text{inside}} = M\,\hat{\mathbf{z}}\cdot\,\hat{\mathbf{r}} = M\cos(\theta) \end{aligned}$$

2. Additional Problem I:

   For a system with cartesian coordinate symmetry, the solution for V has been narrowed down to

$$V(x,y) = \sum_{n=1}^{\infty} C_n \exp\left(-\frac{n\pi x}{a}\right)\,\sin\left(\frac{n\pi}{a}\,y\right)$$

   (a) What are the basis vectors?

   (b) For the boundary condition

$$V(x=5,y) = 3\,\sin\left(\frac{2\pi}{a}\,y\right) - 9\,\sin\left(\frac{7\pi}{a}\,y\right)$$

Determine C_n for all n.

(c) For the different boundary condition (so ignore part (b))

$$V(x = 5, y) = 8$$

write an expression for C_n.